Hadron structure in Lattice QCD

A. Schäfer (Regensburg)

bmb+f - Förderschwerpunkt
Hadronen - und Kernphysik
Großgeräte der physikalischen Grundlagenforschung

- **Introduction:**

⇒ Perturbative QCD, Operator Product Expansion and Lattice QCD
⇒ Chiral perturbation theory and Lattice QCD

- **Examples:**

⇒ hadron and quark masses
⇒ hadron wave function
⇒ Generalized Parton distributions
⇒ topologically non-trivial field configurations.

- **Conclusions**

A disclaimer: I will mainly speak about our lattice results (QCDSF, BGR)

chirally improved fermions	**Clover-Wilson fermions**
hadron spectroscopy	**GPDs, pdf's, FF**
topology	**heavy quark physics**

QCDSF: A. Ali-Khan, D. Broemmel, N. D Cundy, M. Göckeler, Ph. Hägler, T. Hemmert, R. Horsley, Y. Nakamura, M. Ohtani, H. Perlt, D. Pleiter, P.E.L. Rakow, A.S., G. Schierholz, W. Schroers, T. Streuer, H. Stüben, N. Warkentin, V. Weinberg, J. Zanotti, ect.

BGR: F. Bruckmann, T. Burch, D. Chakrabarti, C. Ehmann, C. Gattringer, M. Göckeler, C. Hagen, P. Hasenfratz, D. Hierl, C. B. Lang, M. Limmer, F. Niedermayer, A. Schäfer, S. Solbrig, M. Weingart, etc.

QCD is everything which is calculated from

$$Z[J^a_\mu, \bar{\eta}^i, \eta^i] = \int \mathcal{D}[A^{a\mu}, \bar{\psi}^i, \psi^i] \exp\left(\mathrm{i} \int d^4x \left[\mathcal{L}_{\mathrm{QCD}} - J^a_\mu A^a_\mu - \bar{\psi}^i \eta^i - \bar{\eta}^i \psi^i \right] \right)$$

respectively its low energy limit, which is chiral perturbation theory (ChPT).

A special point of view: Everything else are models. Models are very important and often indispensable to understand what is going on. Still, our aim is to fix all model parameters for hadron structure physics more and more by controlled QCD calculations.

Lattice QCD is one of the essential methods, but still has a long way to go.

In QCD hadron structure is described by correlators of various type

$\left\langle P(p)\middle\vert \bar{q}(x)\gamma_\mu D_{\mu_1}...D_{\mu_n} q(x) \middle\vert P(p)\right\rangle$	**momentum distribution of quarks**
$\left\langle P(p')\middle\vert \bar{q}(x)\gamma_\mu q(x) \middle\vert P(p)\right\rangle$	**form factors of a proton**
$\left\langle P(p)\middle\vert \bar{q}(x)\Gamma_\mu q(x)\bar{q}'(x)\Gamma'_\nu q'(x) \middle\vert P(p)\right\rangle$	**diquark correlations in a proton**
$\left\langle P(p,s)\middle\vert \bar{q}(x)\gamma_\mu \tilde{G}_{\nu\lambda}(x) q(x) \middle\vert P(p,s)\right\rangle$	**color magnetic field in a proton**
$\left\langle 0\middle\vert \bar{d}(-z)\not{z}U(-z,z)u(z) \middle\vert \rho^+(p,s)\right\rangle$	ρ **distribution amplitude**
$\left\langle 0\middle\vert \bar{u}(z)u(z) \middle\vert 0\right\rangle$	**vacuum condensates**

Operator Product Expansion is the art of linking such correlators to physical observables.

A typical example: The spin dependent structure function, $g_1(x,Q^2)$, of the nucleon.

$$g_1(x,Q^2) \stackrel{LO}{=} \frac{1}{2} \sum_f Q_f^2 \left(q_f^{\uparrow}(x,Q^2) - q_f^{\downarrow}(x,Q^2) \right) = \sum_f Q_f^2 \, \Delta q_f(x,Q^2)$$

$$\int_0^1 \Delta q_f(x,Q^2)\, dx = -\frac{S^\sigma}{4M^2} \left\langle PS \middle| \bar{q}_f(x=0)\gamma_\sigma\gamma^5 q_f(x=0) \middle| PS \right\rangle$$

Flavor separated quark distribution functions were extracted by SMC, HERMES, COMPASS, CLAS, **...**

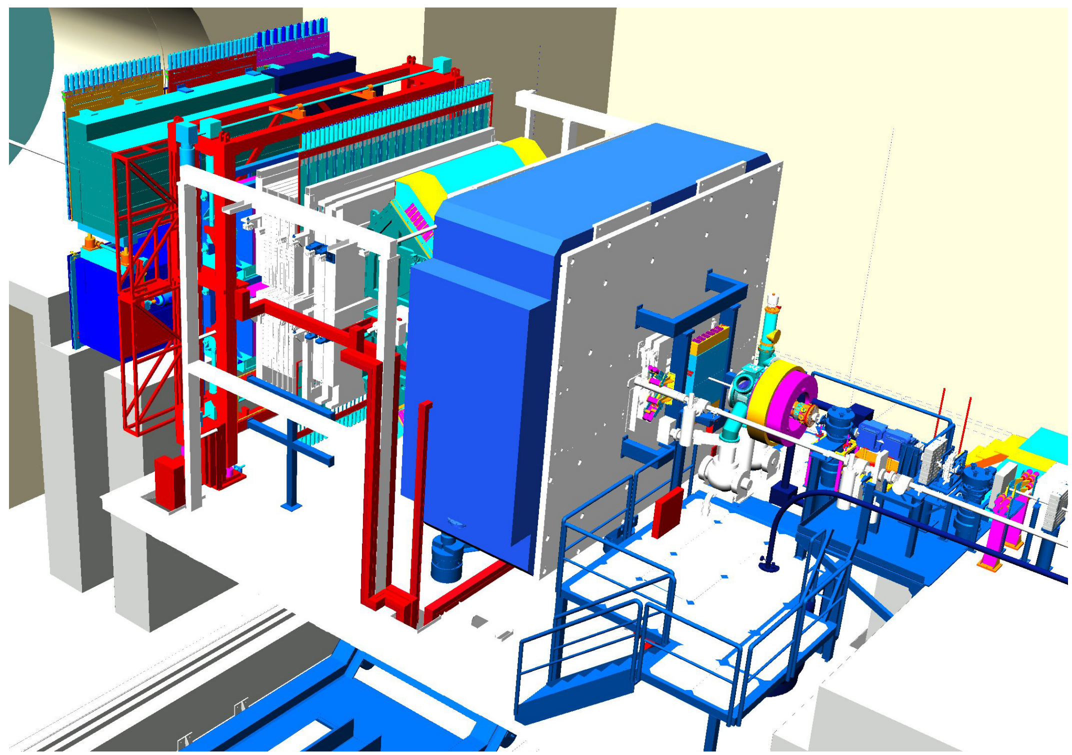
To the extent that lattice results reproduce these, one can, e.g., trust the transverse spin quark distributions in the nucleon $\delta q_f(x,Q^2)$, which are harder to determine experimentally.

However, this is a long way !

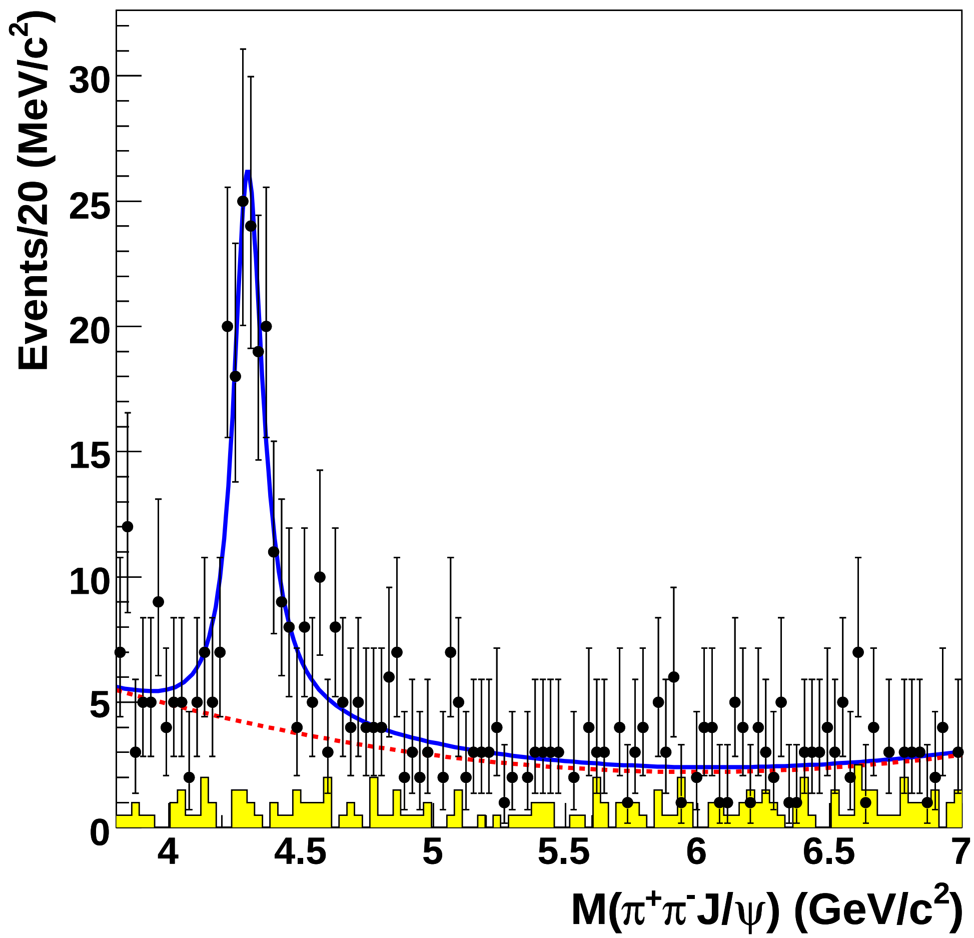
Experiment:

e.g. spin dependent reaction rates $\Rightarrow \Delta u_p(x, Q^2)$

but $\Delta u_p(x, Q^2)$ **is not physical but scheme dependent beyond leading order**



New resonance ⇒ mass and decay width = physical ⇒ much simpler



Invariant mass of $Y(4260)$ candidates from Abe et al. hep-ex/0612006

The message:
Lattice results contain QCD-corrections of arbitrarily high order and can only be compared with a converged pQCD analysis of experimental data.

Often the distinction is made

perturbative QCD	**versus**	**non-perturbative QCD**
high-energy physics	**versus**	**nuclear and hadron physics**

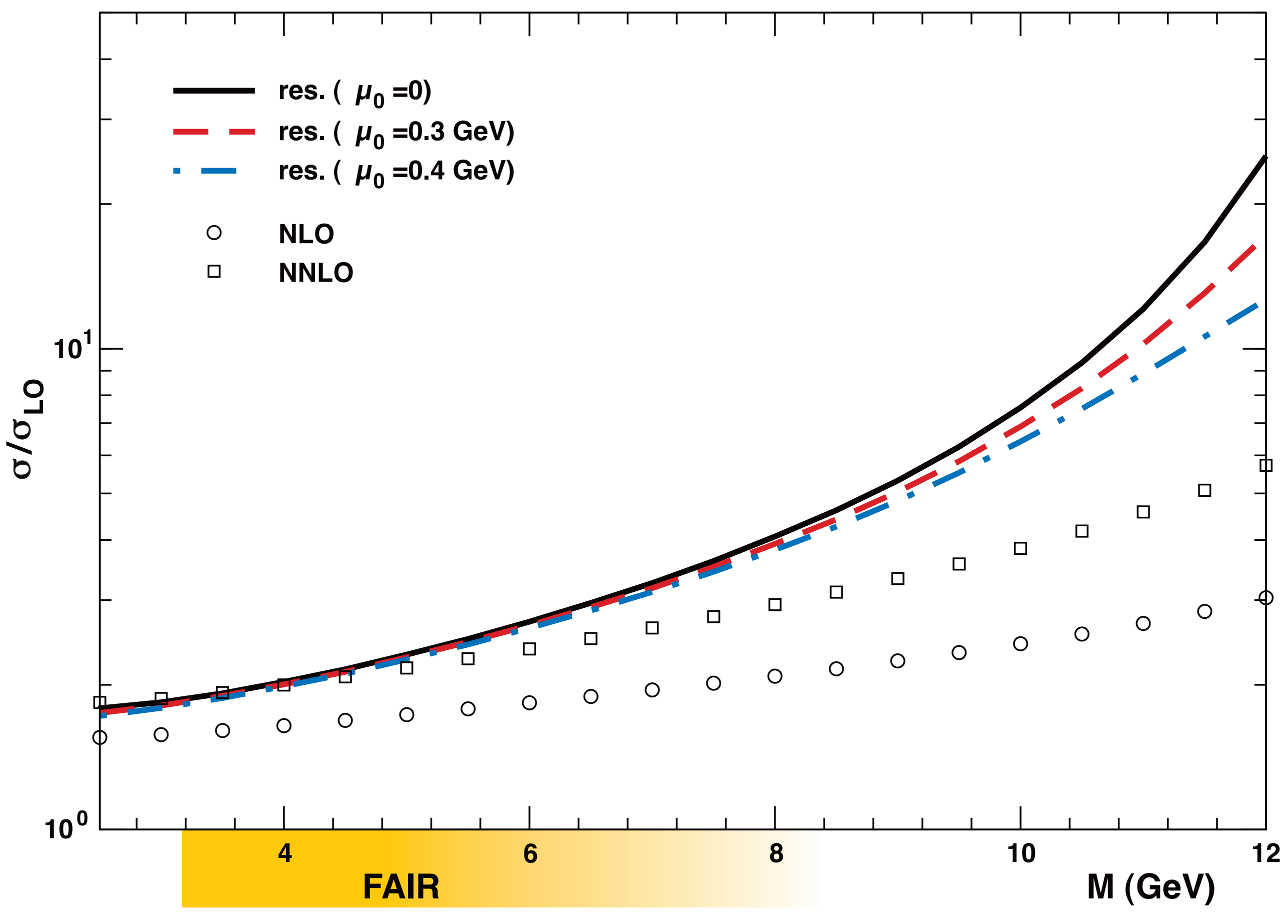
But, to connect, e.g., lattice results for $\Delta q(x;Q^2)$ and experimental results one needs pQCD, and:
- **perturbation theory must converge**
- **factorization theorems, i.e. the twist expansion must converge**

High technical standards are reached: $NLO \rightarrow NNLO$ evolution, resummation, ...), but there might soon be too few people working in that field.

Example: $\delta q_f(x, Q^2)$ from polarized $p + \bar{p}$

A_{TT} at a polarized FAIR collider (??) in the $\overline{MS}$ scheme and NNLO.

H. Shimizu, G. Sterman, W. Vogelsang and H. Yokoya, Phys. Rev. D 71 (2005) 114007



K-factors relative to LO, at $S = 210$ GeV^2.

Lattice QCD: QCD is contained in the generating functional:

$$Z[J^a_\mu, \bar{\eta}^i, \eta^i] = \int \mathcal{D}[A^{a\mu}, \bar{\psi}^i, \psi^i] \exp\left(\mathrm{i} \int d^4x \left[\mathcal{L}_{\mathrm{QCD}} - J^a_\mu A^a_\mu - \bar{\psi}^i \eta^i - \bar{\eta}^i \psi^i\right]\right)$$

You can perform an approximate numerical integration, making use of a relationship between quantum theory and statistical physics:

$$t \quad \leftrightarrow \quad -\mathrm{i}\tau$$

Schrödinger-Equ. $\leftrightarrow$ **Diffusion-Equ.**

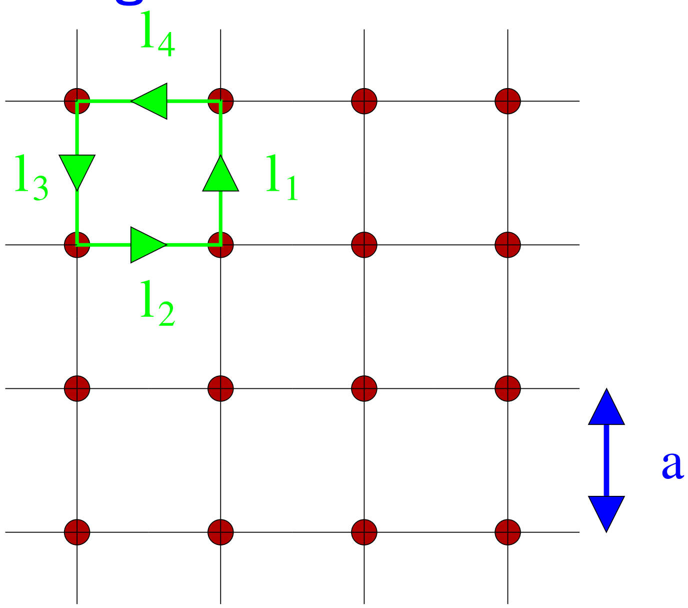
$$-\mathrm{i}\frac{\partial}{\partial t}\psi(\vec{x},t) = \frac{1}{2m} \ \Delta\psi(\vec{x},t) \quad \leftrightarrow \quad \frac{\partial}{\partial t}P(\vec{x},t;\vec{x}_0,t_0) = D \ \ \Delta P(\vec{x},t;\vec{x}_0,t_0)$$

$$S = \int d^4x(T-V) \quad \leftrightarrow \quad \mathrm{i}\int d^4x_E(T+V) = \mathrm{i}S_E$$

$$e^{\mathrm{i}S} \quad \leftrightarrow \quad e^{-S_E}$$

But this trick limits you basically to static quantities (o.k. for hadron structure).

Discretized space time ⇒ e.g. the Wilson action



$$U(l_1) = \exp\left(-igA^b(l_1)\,\frac{\lambda^b}{2}\,a\right)$$

$$W_\square = \mathrm{Tr}\{U(l_1)U(l_2)U(l_3)U(l_4)\}$$

$$\sum_\square \frac{2}{g^2}(3 - \mathcal{R}e\ W_\square) = \frac{1}{4}\int d^4x\,\left(F^a_{\mu\nu}F^a_{\mu\nu}\ +\ O(a^2)\right)$$

Hadronic 2- and 3- Point functions

One needs combinations of field operators (currents), which have the correct quantum numbers, e.g. for the nucleon ($C = i\gamma^2\gamma^4 = C^{-1}$):

$$\hat{B}_\alpha(t,\vec{p}) = \sum_{\vec{x}} e^{\mathrm{i}\vec{p}\cdot\vec{x}} \epsilon_{ijk} \hat{u}^i_\alpha(x)\ \hat{u}^j_\beta(x)(C^{-1}\gamma_5)_{\beta\gamma}\hat{d}^k_\gamma(x)$$

$$\begin{aligned}\langle 0|T\left\{\hat{B}(y_4)\hat{A}(x_4)\right\}|0\rangle &= e^{-(T-y_4+x_4)E_B}\langle B|\hat{B}(0)|0\rangle\langle 0|\hat{A}(0)|B\rangle \\ &+ e^{-(y_4-x_4)E_A}\langle 0|\hat{B}(0)|A\rangle\langle A|\hat{A}(0)|0\rangle\end{aligned}$$

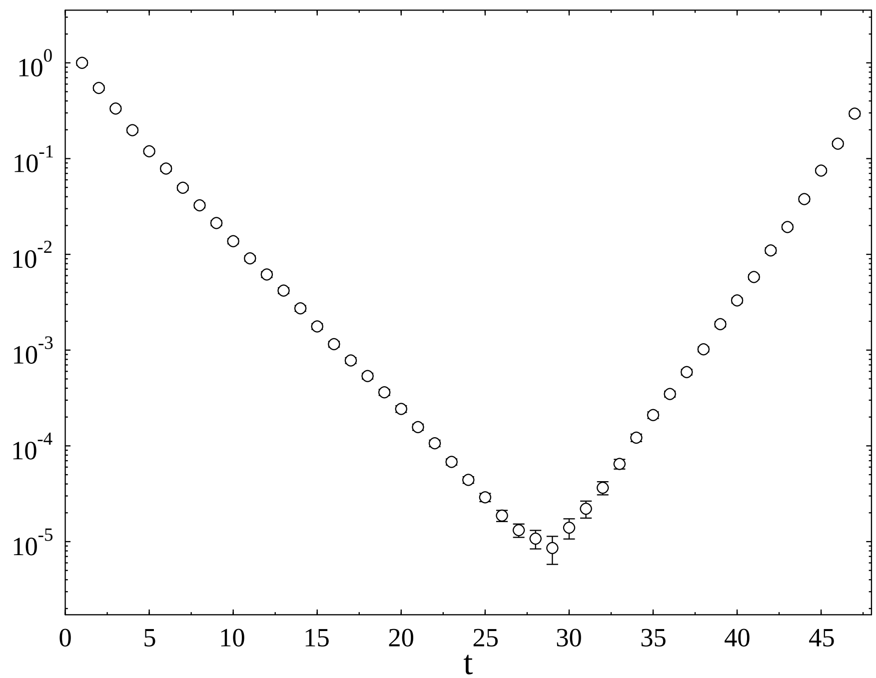
$\hat{B}$ **generates the antiparticle of** $\hat{A}$**.**

To get the hadron masses one simply has to determine the slopes.

$$e^{-(y_4-x_4)M_N}\langle 0|\hat{N}^\dagger(0)|N\rangle\langle N|\hat{N}(0)|0\rangle$$

$$\begin{aligned}|B\rangle \quad &\sim \quad c_0|N\rangle \;+\; c_1|N'\rangle \;+\; c_2|N\pi\rangle \;+\; \ldots \\ &\Rightarrow \quad c_0 e^{-E_N t}|N\rangle \;+\; c_1 e^{-E_{N'} t}|N'\rangle \;+\; c_2 e^{-E_{N\pi} t}|N\pi\rangle \;+\; \ldots\end{aligned}$$

$$\langle B_\alpha(t,\vec{p})\bar{B}_\beta(0,\vec{p})\rangle$$



Once the propagation in imaginary time has projected the original source onto the physical wave function on can calculate physical correlators from

$$\frac{\tilde{\Gamma}_{\alpha\beta}\langle B_\beta(t,\vec{p})\mathcal{O}\bar{B}_\alpha(0,\vec{p})\rangle}{\Gamma_{\alpha\beta}\langle B_\beta(t,\vec{p})\bar{B}_\alpha(0,\vec{p})\rangle}$$

For non-zero momentum transfer all normalization factors and exponentials cancel in the ratio R.

$$R_{\mathcal{O}}(\tau,P_2,P_1) = \frac{C^{\textbf{3pt}}_{\mathcal{O}}(\tau,P_2,P_1)}{C^{\textbf{2pt}}(\tau_{\textbf{snk}},P_2)}\left[\frac{C^{\textbf{2pt}}(\tau,P_2)C^{\textbf{2pt}}(\tau_{\textbf{snk}},P_2)C^{\textbf{2pt}}(\tau_{\textbf{snk}}-\tau+\tau_{\textbf{src}},P_1)}{C^{\textbf{2pt}}(\tau,P_1)C^{\textbf{2pt}}(\tau_{\textbf{snk}},P_1)C^{\textbf{2pt}}(\tau_{\textbf{snk}}-\tau+\tau_{\textbf{src}},P_2)}\right]^{\frac{1}{2}}$$

$$C^{\textbf{2pt}}(\tau,P_1) = \sum_{j,k}\tilde{\Gamma}_{jk}\left\langle N_k(\tau,P_1)\overline{N}_j(\tau_{\textbf{src}},P_1)\right\rangle$$

$$C^{\textbf{3pt}\,\mu\nu\mu_1\ldots\mu_{n-1}}_{\mathcal{O}}(\tau,P_2,P_1) = \sum_{j,k}\tilde{\Gamma}_{jk}\left\langle N_k(\tau_{\textbf{snk}},P_2)\mathcal{O}_T^{\mu\nu\mu_1\ldots\mu_{n-1}}(\tau)\overline{N}_j(\tau_{\textbf{src}},P_1)\right\rangle$$

The problem of renormalization

lattice Dirac operator	$\neq$	continuum Dirac operator
lattice propagators	$\neq$	continuum propagators
lattice renormalization	$\neq$	continuum renormalization

$\Rightarrow$ additional lattice renormalization factors are needed to get continuum results

Also, on the lattice you have a hyper-cubic symmetry group

$$H(4) \ll \text{ the Lorentz group}$$

$\Rightarrow$ lattice OPE $\neq$ continuum OPE

In a perfect world you would like to have:

- **a** $\rightarrow 0$ **continuum limit**
- $V = L^4 \rightarrow \infty$ **infinite volume limit**
- $m_q \rightarrow m_q(\text{physical})$ **chiral/physical limit**
- **good chiral symmetry**
- **infinitely many uncorrelated field configurations**

Different groups choose different compromises $\Rightarrow$ **Heated discussions.**

Options: Overlap fermions, domain wall-fermions, Chirally improved fermions, perfect action fermions, twisted mass fermions, FLIC fermions, Wilson fermions, staggered fermions, ... and various mixtures, as well as the suitable gauge actions.

Warning: The reliability of lattice results is not reflected by just the statistical error bars or the values of m_π **cited, but depends also on the quality of the used lattice formulation.**

Chiral Perturbation Theory:

The correct treatment of ChPT (there do exist many versions) is controversially debated, but hardly anybody questions that combining ChPT and lattice QCD is crucial.

Presently, the range of momenta, volumes and m_π for which ChPT is well under control hardly overlaps with those of lattice simulations.

But already now ChPT improves substantially the $a \to 0$ and $V \to \infty$ limits, and lattice QCD fixes low energy constants of ChPT.

Two-point functions: Masses

Ground state masses can be reproduced meanwhile with high accuracy, so the interest shifted towards more specific questions, e.g.

- **the extraction of quark masses**

$$\bar{m}_s(\mu = 2GeV) \; = \; f\Big(m_{K^+}, m_{K^0}, m_{\pi^+}\Big)$$

where the function f is given by ChPT.

M. Göckler et al. [QCDSF] hep-lat/061007:

$$\bar{m}_s(\mu = 2GeV) \; = \; 115(2)(3)(6) \;\; \mathrm{MeV}$$

while PDG gives (without lattice):

$$\bar{m}_s(\mu = 2GeV) \; = \; 103 \pm 20 \;\; \mathrm{MeV}$$

(Note that other lattice groups get somewhat different values.)

Resonance masses

- **quenched chiral fermions:**

We use the generalized eigenvalue method of Michael, Lüscher and Wolf:

$$C(t)_{ij} \;=\; \sum_n \langle\, 0 \,|\, O_i \,|\, n \,\rangle \langle\, n \,|\, O_j^\dagger \,|\, 0 \,\rangle \, e^{-t\, M_n} \; .$$

$$C(t)\, \vec{v}^{(k)} \;=\; \lambda^{(k)}(t)\, C(t_0)\, \vec{v}^{(k)} \qquad , \qquad \lambda^{(k)}(t) \;\propto\; e^{-t\, M_k} \,[\, 1 + \mathcal{O}(e^{-t\, \Delta M_k})\,] \; ,$$

and a variety of sources

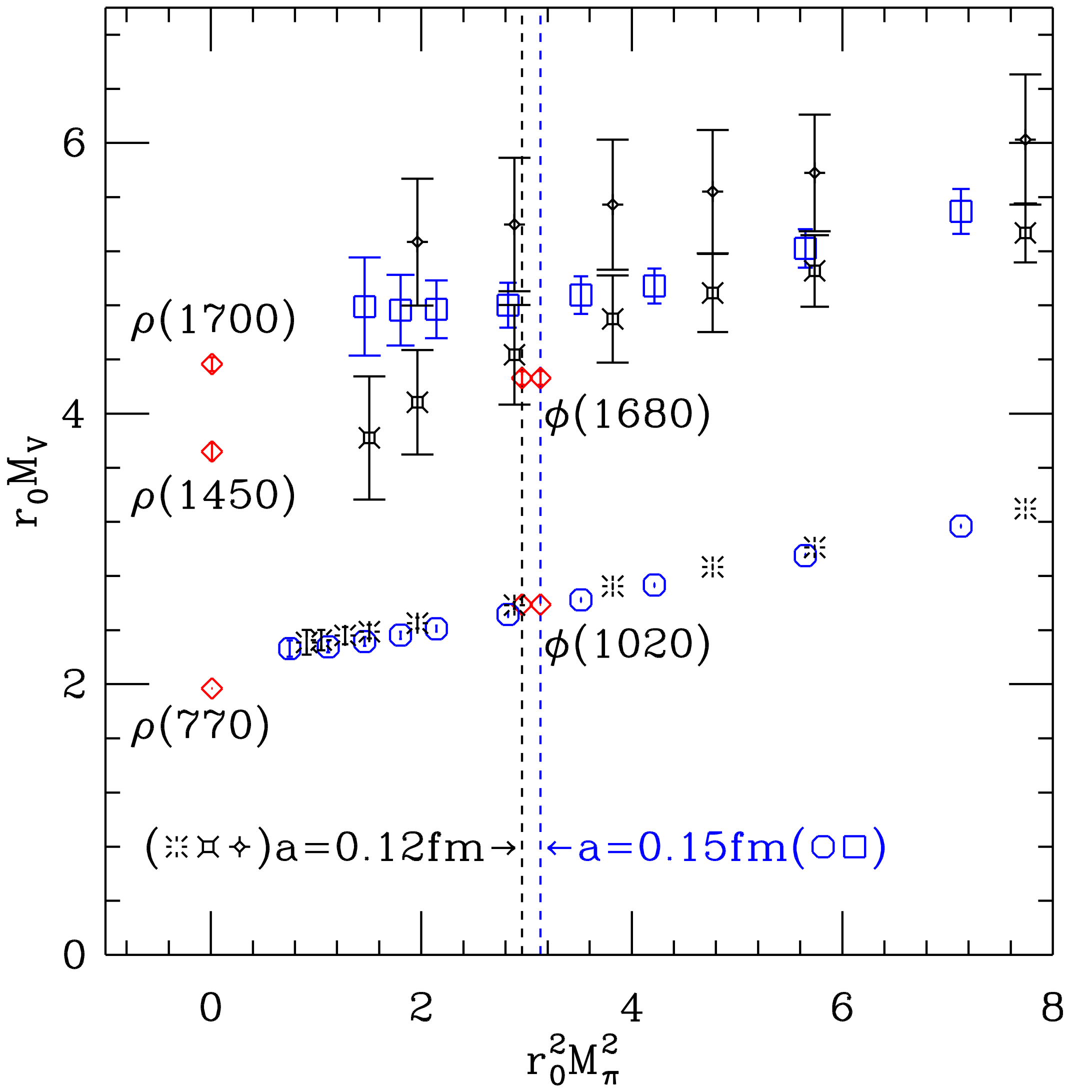
$$\mathcal{O}_i \;=\; \bar{q}_f(x) \Gamma q_{f'}(x)$$

smeared to different extend by acting with

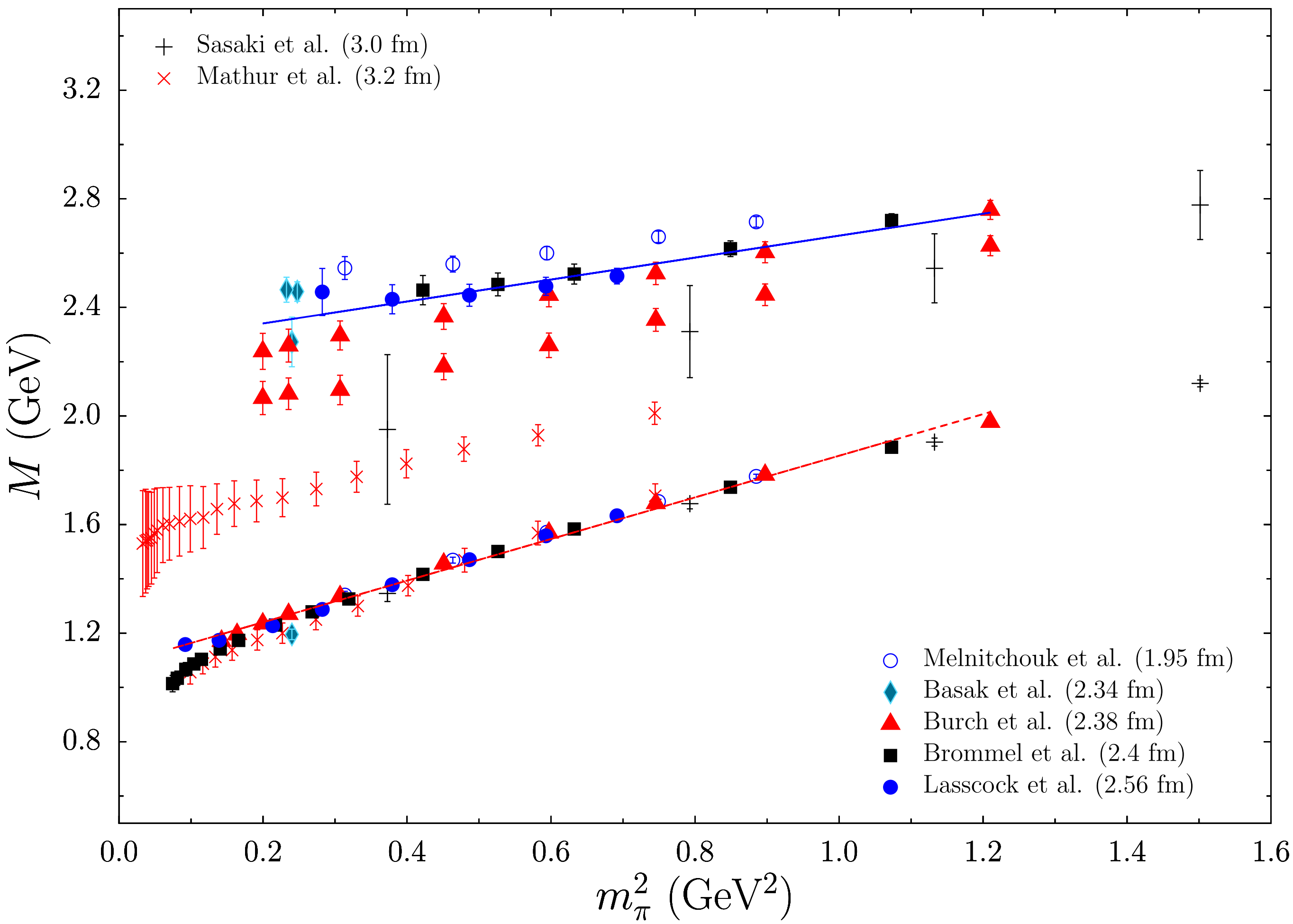
$$M \;=\; \sum_{n=0}^{N} \kappa^n\, H^n \; ,$$

$$H(\vec{x}, \vec{y}\,) \;=\; \sum_{j=1}^{3} \Big[\, U_j(\vec{x}, 0)\, \delta(\vec{x} + \hat{j}\,, \vec{y}\,) \;+\; U_j(\vec{x} - \hat{j}\,, 0)^\dagger\, \delta(\vec{x} - \hat{j}\,, \vec{y}\,) \Big] \, .$$

The results look typically nice for mesons [BGR]:

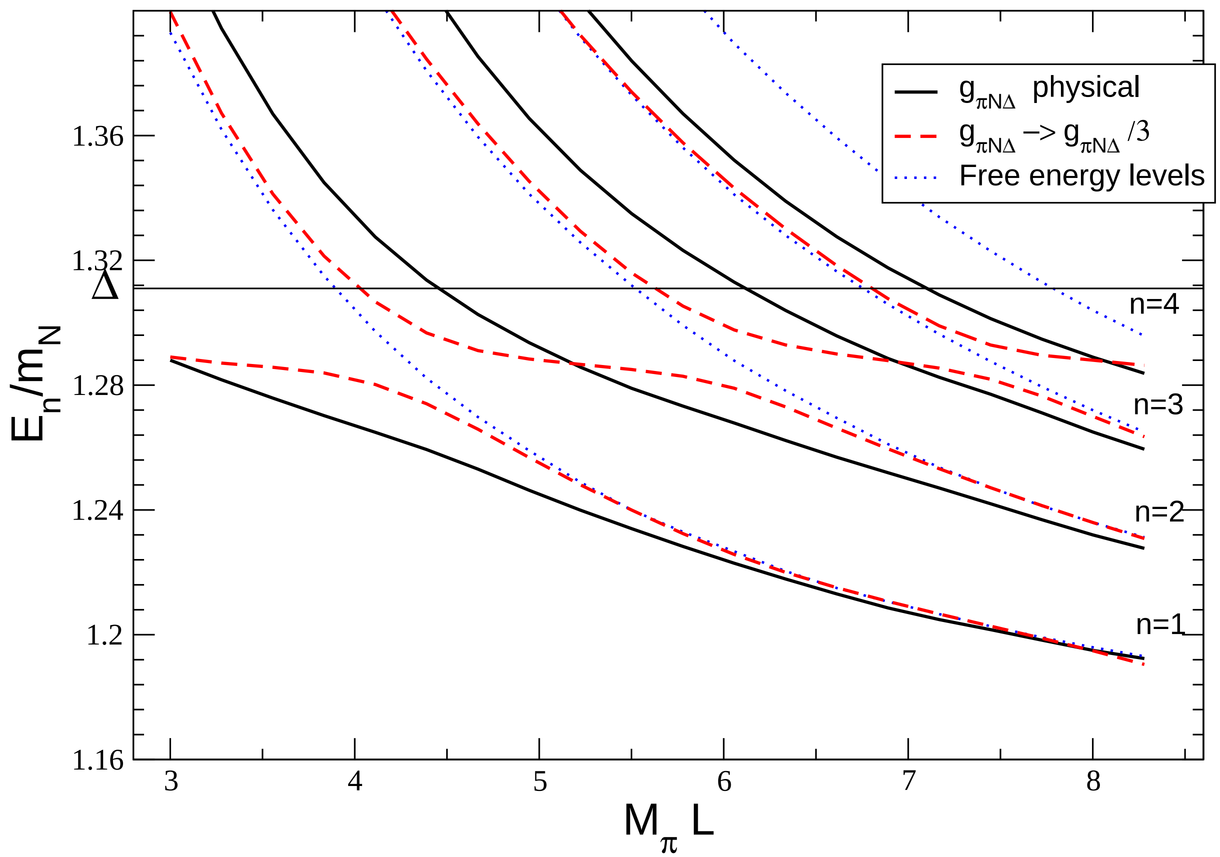


and problematic for baryons, here N:



Sasaki et al. and Mathur et al. use Bayesian techniques, the rest correlation matrix techniques, from B. Lasscock et al., hep-lat/0705.0861.

• **dynamical (chiral) fermions: Calculations are just starting. The continuum is discrete on the lattice**
⇒ **avoided level crossings**
⇒ **mass and width of the resonance**
Problem: hadronic corrections are large
⇒ **one needs again a combination of ChPT and Lattice-QCD**
V. Bernard, U. G. Meissner and A. Rusetsky, hep-lat/0702012.



Two-point functions: Wave functions

$$\phi_\Pi(x,\mu^2) = Z_2(\mu^2)\int^{|k_\perp|<\mu} d^2k_\perp\, \phi_{\Pi,BS}(x,k_\perp).$$

with the Bethe -Salpeter wave function $\phi_{\Pi,BS}$.

$$\langle 0|\bar{q}(-z)\gamma_\mu\gamma_5 U(-z,z)u(z)|\Pi^+(p)\rangle = if_\Pi p_\mu \int_{-1}^{1} d\xi\, e^{-i\xi p\cdot z}\phi_\Pi(\xi,\mu^2)\,,$$

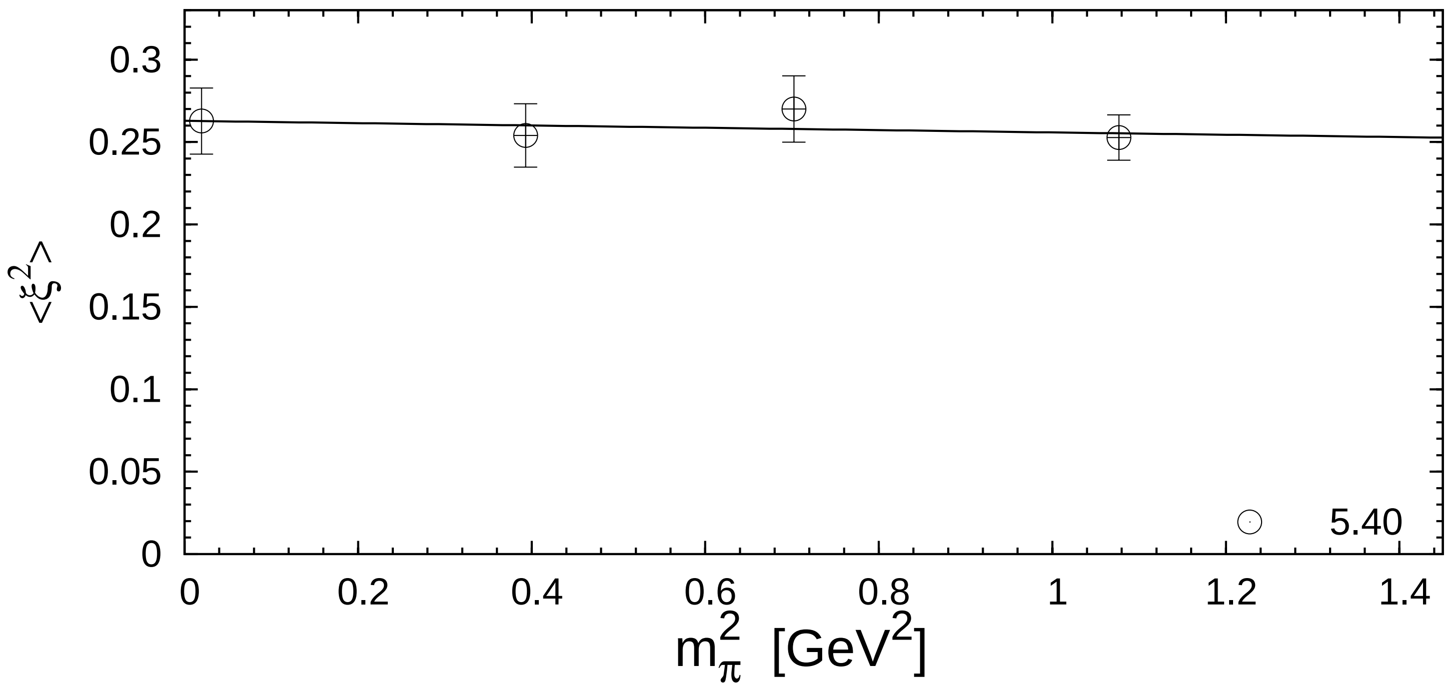
$$\phi_\Pi(\xi,\mu^2) = \frac{3}{4}(1-\xi^2)\left(1+\sum_{n=1}^{\infty} a_n^\Pi(\mu^2)C_n^{3/2}(\xi)\right).$$

matrix elements of

$$\mathcal{O}_{\{\mu_0\ldots\mu_n\}} = \bar{q}\gamma_{\{\mu_0}\gamma_5 \overleftrightarrow{D}_{\mu_1}\ldots\overleftrightarrow{D}_{\mu_n\}}u\,,$$

give ξ^n-moments

Braun et al. [QCDSF] hep-lat/0606012 $N_f = 2$ **improved Wilson fermions**



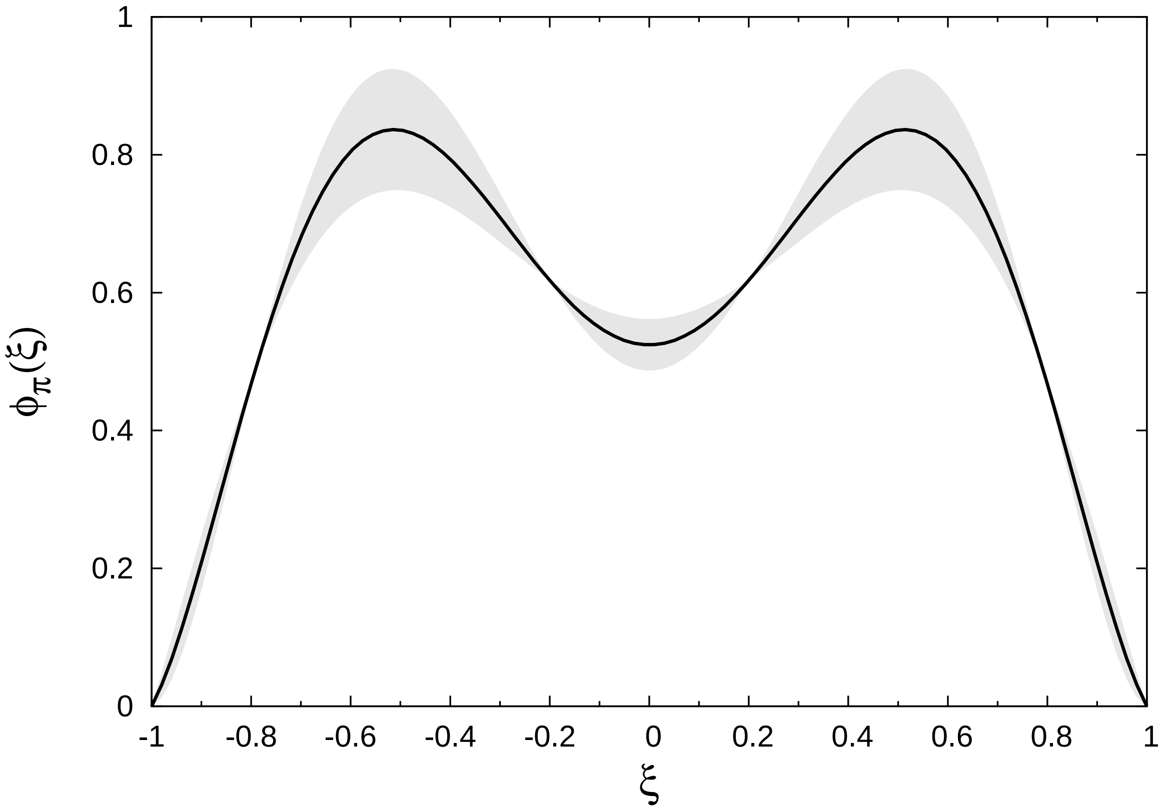
Chiral extrapolation of $\langle \xi^2 \rangle_\pi$ **at for** $\beta = 5.40$ **for** $\mathcal{O}^a_{412}$ **in the** $\overline{\mathrm{MS}}$ **scheme at** $\mu^2 = 4\,$**GeV**^{2}**.**

result

$$a_2^\pi(\mu^2 = 4\ \mathrm{GeV}^2) = 0.201(114)$$

QCD sumrules, B-decays and transition formfactors:

$$a_2^\pi(4\ \mathrm{GeV}^2) = 0.17 \pm 0.15$$



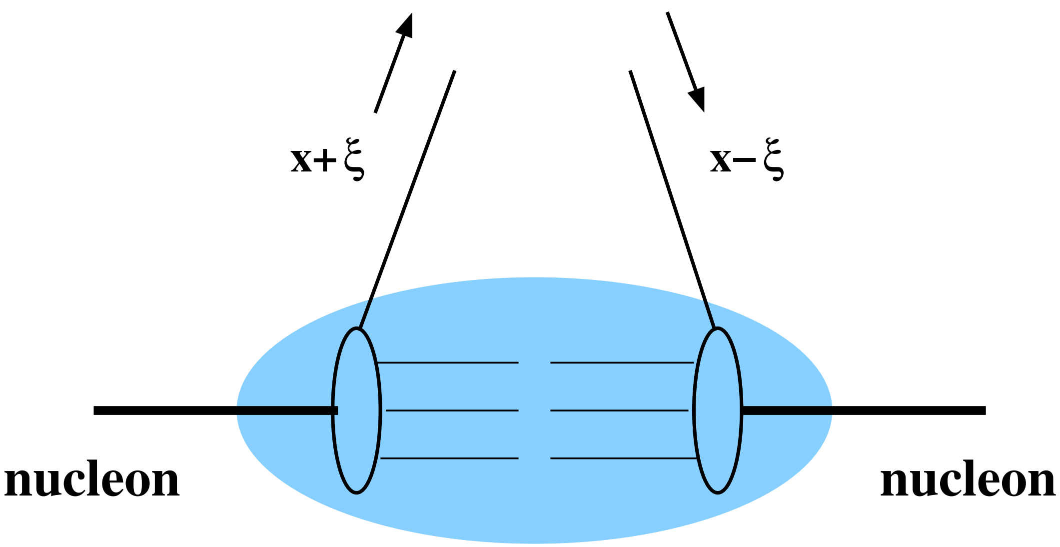
Three-point functions: Generalized Parton Distributions

GPDs are related to generalized matrix elements of the form

$$\Big\langle P(p,s)\Big|\bar{q}(-z)\gamma_\mu U(-z,z)q(z)\Big|P(p',s')\Big\rangle$$

which can be treated with the same rigor.

GPDs are a set of functions H, E, $\tilde{H}$, $\tilde{E}$ H_T, E_T, $\tilde{H}_T$ and $\tilde{E}_T$, each depending on the three variables x, ξ, t,



$$P = (p+p')/2 \quad , \quad \Delta = p - p'$$

GPDs contain all distribution functions, form factors etc. as limiting cases, e.g.,

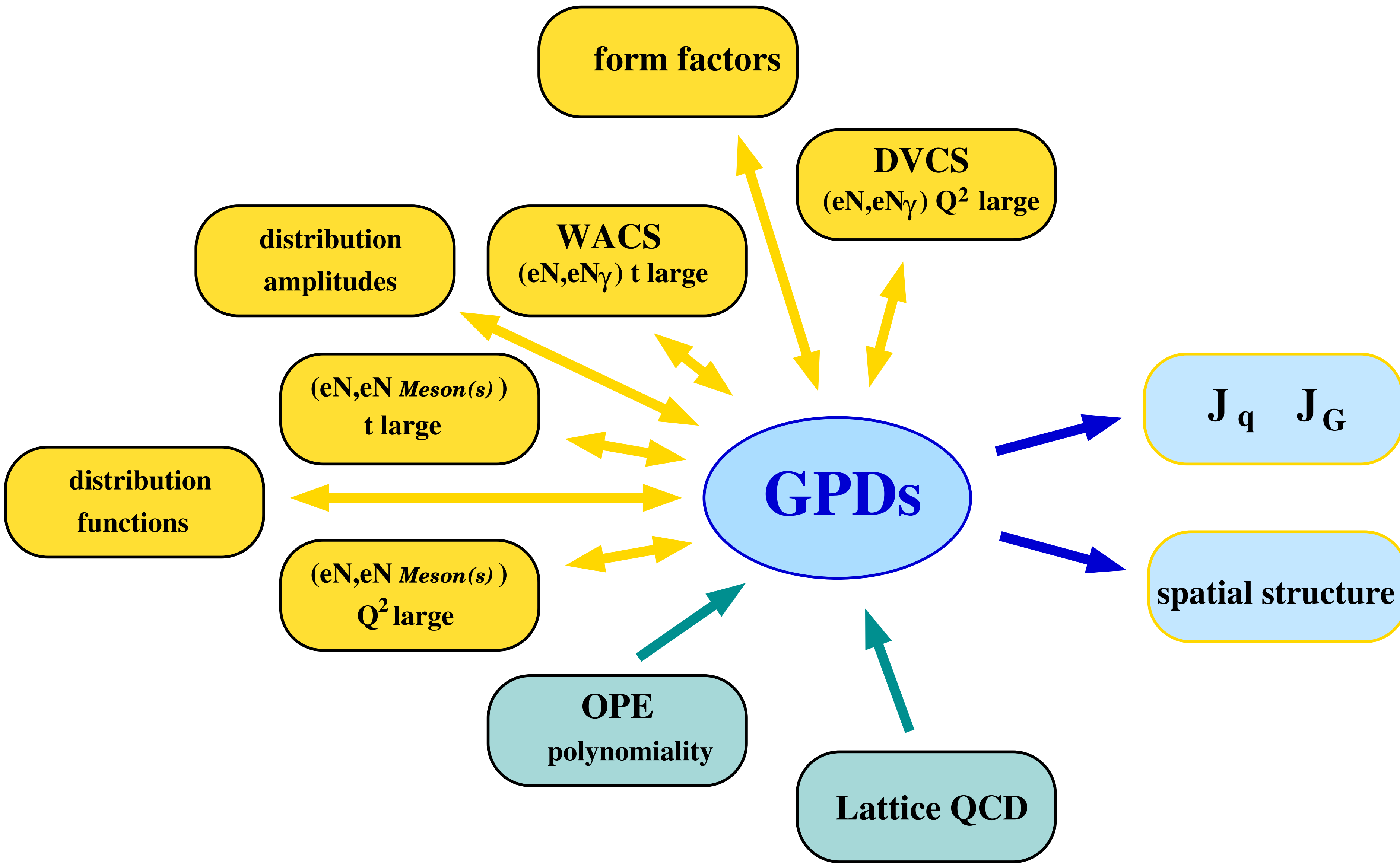
$$H_q(x,0,0) = q(x) \qquad \int_{-1}^{1} dx H_q(x,\xi,t) = F_{1q}(t)$$

$$\tilde{H}_q(x,0,0) = \Delta q(x) \qquad \int_{-1}^{1} dx H_q(x,\xi,t) = g_{Aq}(t)$$

GPDs give information on the transverse structure of hadrons in the impact parameter plane.
The transverse mass is $\sqrt{q_{\|}^2 + m^2}$. Therefore a probabilistic interpretation makes sense.

$$H_q(x,0,\mathbf{b}_\perp^2) = \frac{1}{(2\pi)^2} \int d^2\boldsymbol{\Delta}_\perp \, \mathbf{e}^{\mathbf{i}\mathbf{b}_\perp \boldsymbol{\Delta}_\perp} H_q(x,0,\boldsymbol{\Delta}_\perp^2)$$

All of the different GPDs have a very specific phenomenological meaning.


form factors
DVCS
(eN,eNγ) Q² large
distribution
amplitudes
WACS
(eN,eNγ) t large
(eN,eN Meson(s))
t large
J q J G
distribution
functions
GPDs
(eN,eN Meson(s))
Q² large
spatial structure
OPE
polynomiality
Lattice QCD

The operator matrix elements are parameterized as follows:

$$\mathcal{O}_q^{\mu\mu_1\cdots\mu_n} := \textbf{Sym}\ \bar{q}(x)\gamma^\mu \mathrm{i}\overleftrightarrow{D}^{\mu_1} \ldots \mathrm{i}\overleftrightarrow{D}^{\mu_n}\ q(x)$$ local operators

$$\left\langle P_2 \middle| \mathcal{O}_q^{\mu\mu_1\cdots\mu_n} \middle| P_1 \right\rangle = \textbf{Sym}\ \bar{N}(P_2)\gamma^\mu N(P_1) \sum_{i=0,even}^{n} A^q_{n+1,i}(t)\Delta^{\mu_1}\ldots\Delta^{\mu_i}P^{\mu_{i+1}}\ldots P^{\mu_n}$$

$$+ \textbf{Sym}\ \bar{N}(P_2)\frac{\mathrm{i}\sigma^{\mu\alpha}\Delta_\alpha}{2M}N(P_1) \sum_{i=0,even}^{n} B^q_{n+1,i}(t)\Delta^{\mu_1}\ldots\Delta^{\mu_i}P^{\mu_{i+1}}\ldots P^{\mu_n}$$

$$+ \textbf{Sym}\ \bar{N}(P_2)\frac{\Delta_\mu}{M}N(P_1)\ C^q_{n+1}(t)\mathrm{mod}(n,2)\Delta^{\mu_1}\ldots\Delta^{\mu_n}$$

$$\int_{-1}^{1} dx\, x^{n-1}\, H(x,\xi,t) \quad = \quad \sum_{\substack{k=0 \\ \text{even}}}^{n-1} (2\xi)^k\, A_{n,k}(t) + \text{mod}(n+1,2)\,(2\xi)^n C_n(t)$$

$$\int_{-1}^{1} dx\, x^{n-1}\, E(x,\xi,t) \quad = \quad \sum_{\substack{k=0 \\ \text{even}}}^{n-1} (2\xi)^k\, B_{n,k}(t) - \text{mod}(n+1,2)\,(2\xi)^n C_n(t)$$

A famous relation: Ji's sumrule

$$\left\langle J_q^3 \right\rangle \;=\; \frac{1}{2}[A_{2,0}^q(0) + B_{2,0}^q(0)]$$

The following moments have so far been calculated on the lattice

N:

A^q_{10}, A^q_{20}, A^q_{30}, A^q_{32}, B^q_{10}, B^q_{20}, B^q_{30}, B^q_{32}, C^q_{20},

$\tilde{A}^q_{10}$, $\tilde{A}^q_{20}$, $\tilde{A}^q_{30}$, $\tilde{A}^q_{32}$, $\tilde{B}^q_{10}$, $\tilde{B}^q_{20}$, $\tilde{B}^q_{30}$, $\tilde{B}^q_{32}$,

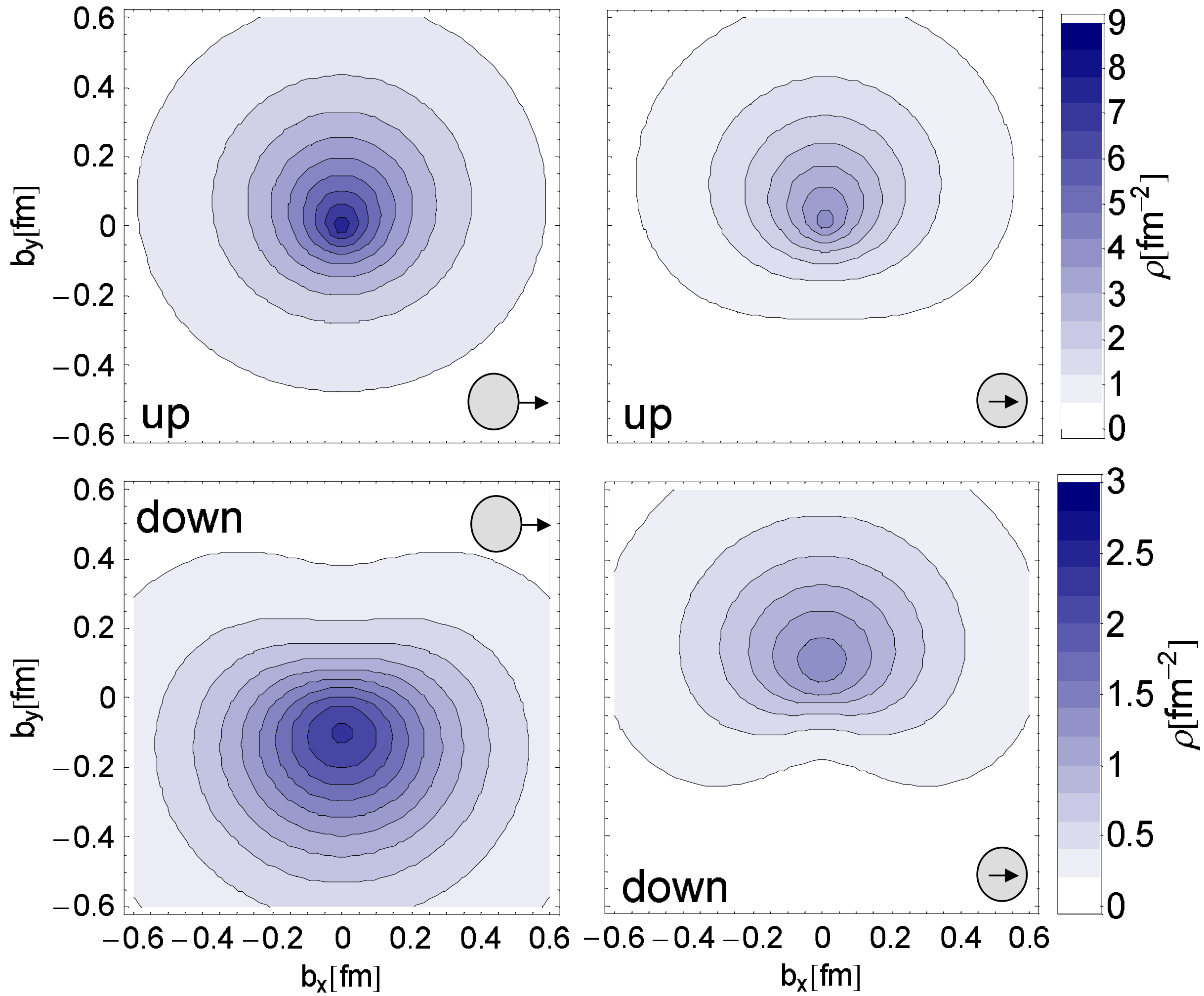
A^q_{T10}, A^q_{T20}, $\bar{B}^q_{T10} = B^q_{T10} + 2\tilde{A}^q_{T10}$, $\bar{B}^q_{T20}$, $\tilde{A}^q_{T10}$, $\tilde{A}^q_{T20}$, $\tilde{B}^q_{T21}$

π**:**

A^q_{10}, A^q_{20}, C^q_{20}, B^q_{T10}, B^q_{T20}

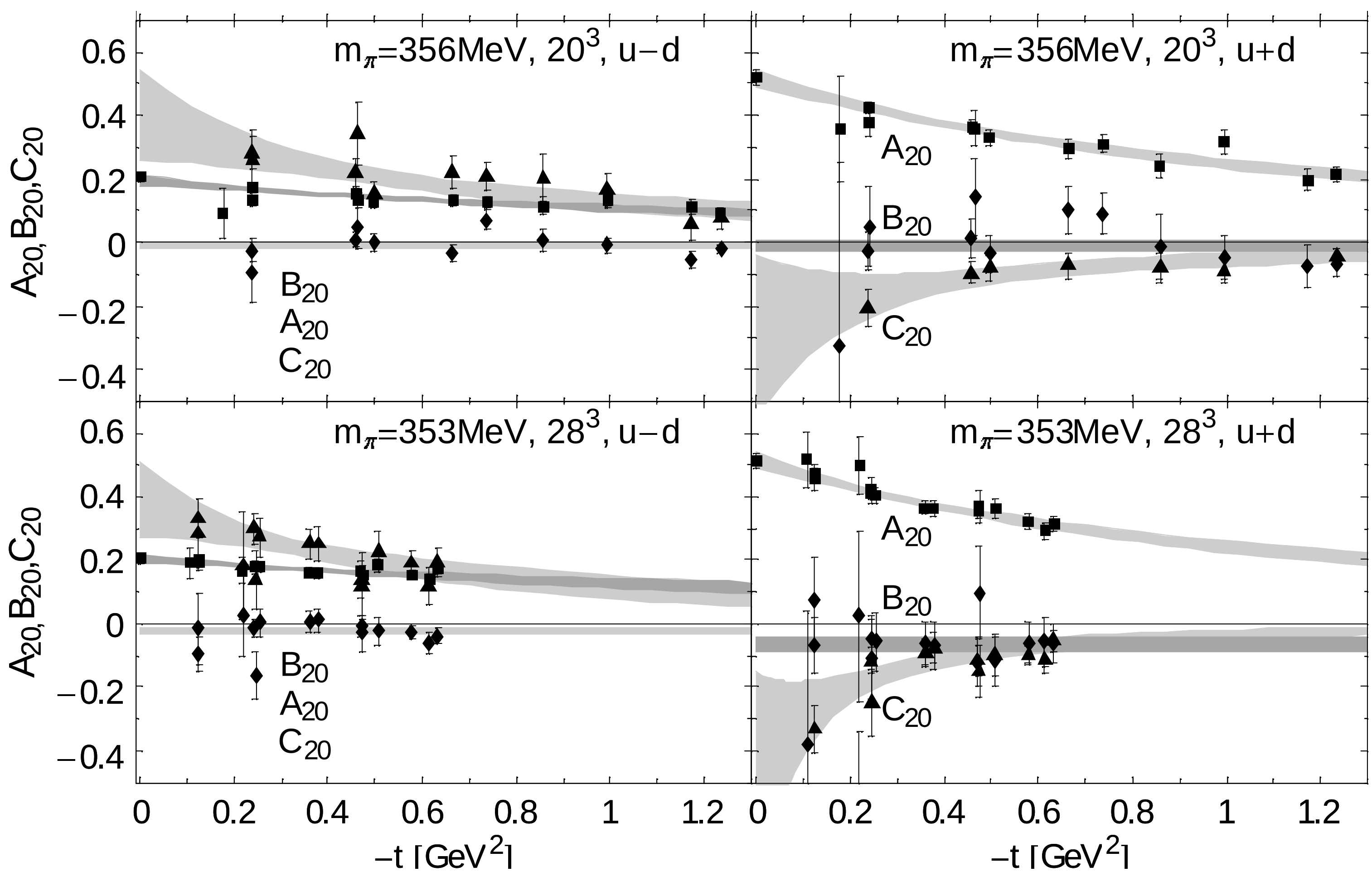
I will only discuss a few examples.

A_{10}, A_{20} $\Rightarrow$ **transverse quark distribution**
plus A_{T10}, $\tilde{A}_{T10}$, $\overline{B}_{T10}$ $\Rightarrow$ **transverse quark distribution in a transversely polarized nucleon**
M. Göckeler et al., hep-lat/0612032



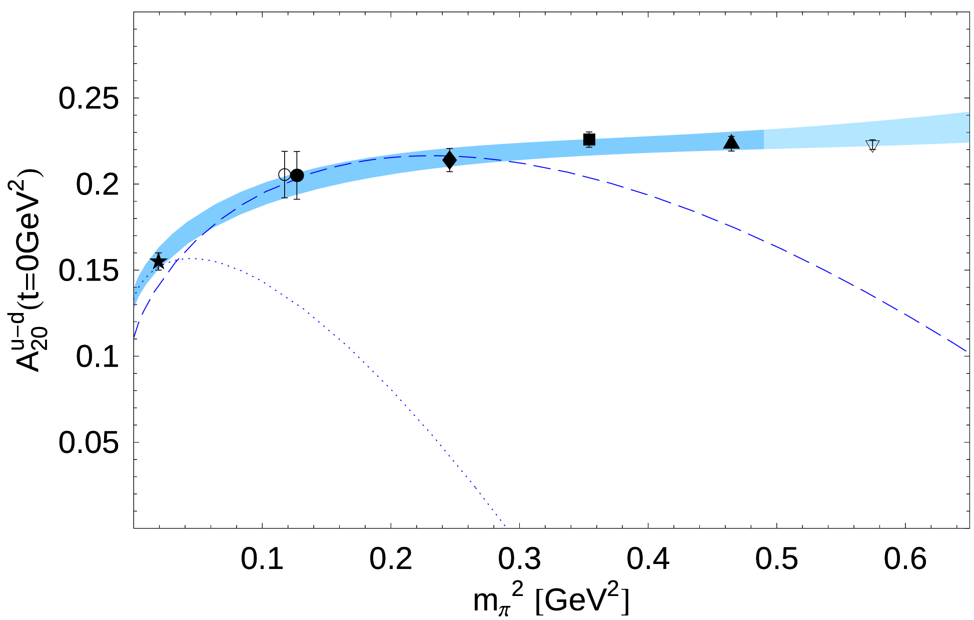
Ji's sumrule A_{20}, $B_{20} \Rightarrow J_u$, J_d

P. Hägler et al. [LHPC], hep-lat/0705.4295



Chiral extrapolation to the physical pion mass with CBChPT, see
M. Dorati, T. Gail and T. Hemmert, nucl-th/0703073

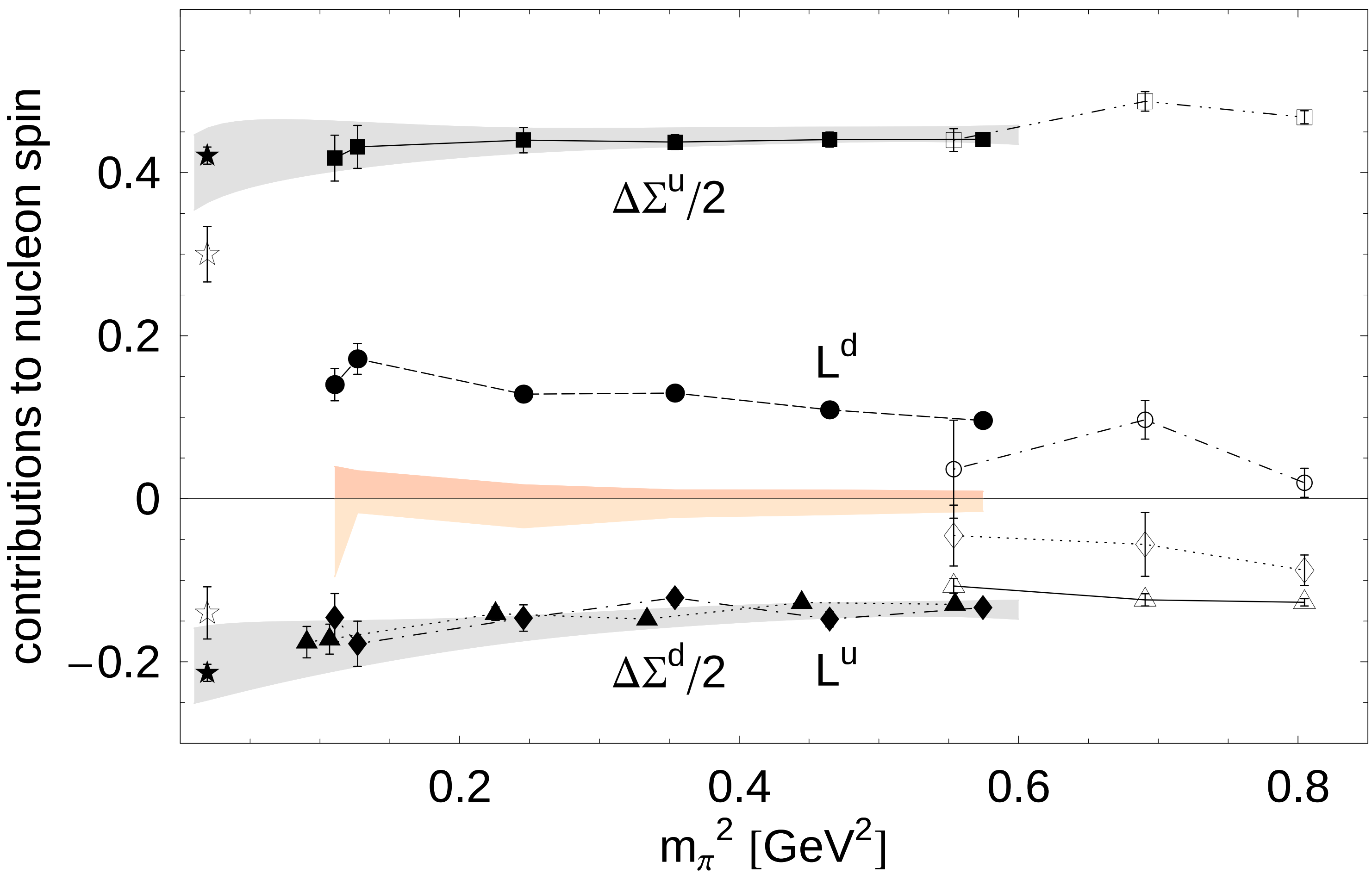
$$A_{20}^{u-d}(t,m_\pi) = A_{20}^{0,u-d}\left(f_A^{u-d}(m_\pi) + \frac{g_A^2}{192\pi^2 f_\pi^2} h_A(t,m_\pi)\right) + \widetilde{A}_{20}^{0,u-d} j_A^{u-d}(m_\pi) + A_{20}^{m_\pi,u-d} m_\pi^2 + A_{20}^{t,u-d} t$$



Error band: CBChPT

dashed line: HBChPT

dotted line: heavy baryon limit of CBChPT

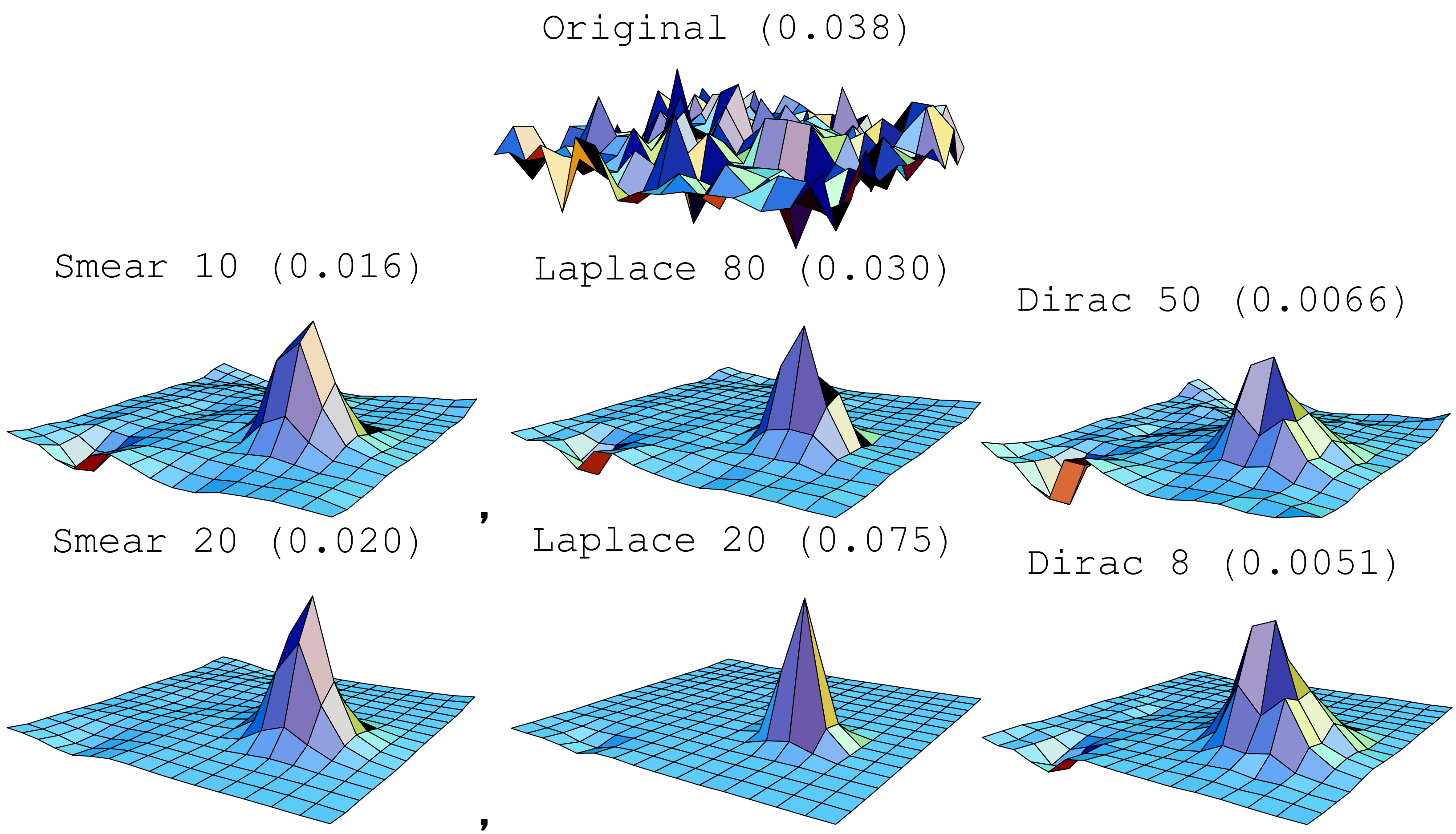


Quark spin and orbital angular momentum contributions to the spin of the nucleon for up and down quarks.

Filled stars: HERMES 2007

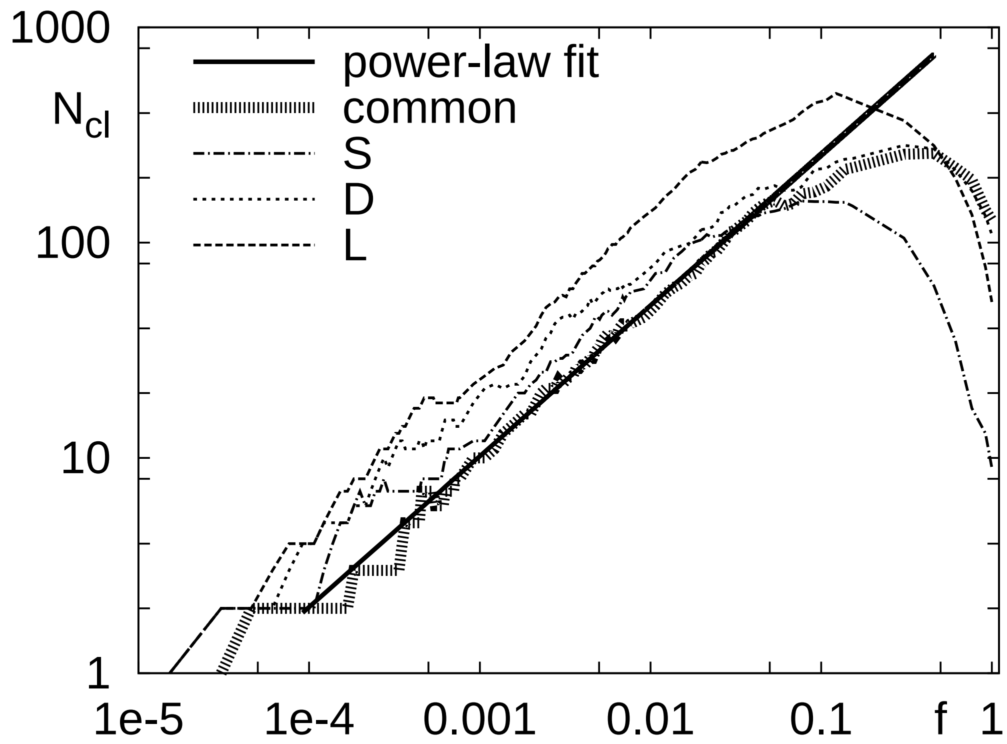
Open lattice symbols: LHPC/SESAM

Topologically non-trivial field configurations



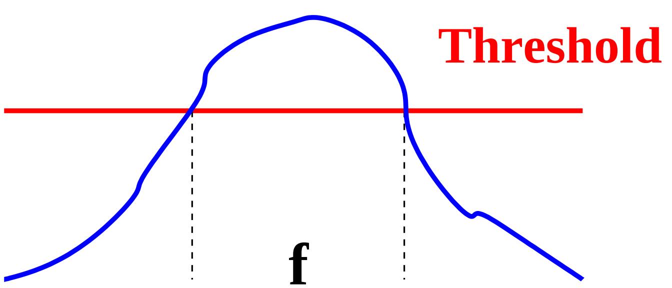
Different filtering methods give similar results, when handled with care.

combining them gives cleaner results, e.g. a new power law.



number of topological charge clusters with volume fraction f

topological charge density



Dilute instanton gas: exponent $\xi = 5/11$

lattice: $\xi = 0.59(5)$

Conclusions

• **Lattice QCD allows to study many different aspects of hadron structure. I could only present some very few examples.**

• **To reach high precision results, one needs the combination**
Lattice QCD & pQCD & ChPT

• **Lattice QCD evolves extremely fast, powered by:**
• **Moore's law**
• **a Moore*'s law for algorithms**
• **analytic work relating lattice observables to physics.**

• **Models will never be superfluous, as lattice QCD can only address a limited range of questions, but models should not disagree with lattice results, where the latter are reliable.**