FEW-BODY *AB INITIO* SCATTERING CALCULATIONS INCLUDING COULOMB

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***AB INITIO* CALCULATIONS** of 3N and 4N reactions, together with structure calculations up to $\simeq$ mass 12, have been up to now the natural "theoretical laboratory" to test the force models between hadrons.

Present models are based on

$$\begin{cases} \text{meson theory (AV18, CD-Bonn, CD- Bonn} + \Delta's\ldots) \\ \text{chiral perturbation theory (Machleidt, Epelbaum and collaborators)} \end{cases}$$

All potentials are fitted to the 2N data up to π production threshold. χ^2/datum can be as low as $\simeq 1$.

3N forces, though essential to describe the energy levels of nuclei, play a negligible role in 3N scattering at low energy (except through scaling). What happens in 4N scattering?

DIRECT NUCLEAR REACTION: Few-body degrees of freedom play an important role in nuclear reactions driven by deuterons or halo nuclei where exact three-body calculations have been lacking for fifty years.

Only DWBA, Adiabatic and CDCC, which are approximate methods, have prevailed as the tool to analyse data, and extract structure information on specific nuclear states.

TOPICS

1. **Three-nucleon reactions with two-protons**

   A. Deltuva, A.C. Fonseca and P. Sauer

2. **Four-nucleon reactions with two-, or three-protons**

   A. Deltuva and A.C. Fonseca

3. **Three-body nuclear reactions involving the scattering of deuterons (halo nuclei) on a heavy (light) target**

   A. Deltuva, A. Moro, E. Cravo, F.M. Nunes and A.C. Fonseca

1. THREE-NUCLEON REACTIONS WITH TWO-PROTONS

$p + d$ (instead of $n + d$)

Until 2005 most elastic and breakup $p + d$ data was analysed with $n + d$ calculations where the Coulomb force is absent.

Exceptions: pd $\rightarrow$ pd and pd $\leftrightarrow$ γ^3He at $E_{CM} < 20$ MeV and local potentials using Kohn's Variational Principle plus Hyperspherical Harmonics (Pisa Group)

Now we can calculate all 3N reactions for 1 MeV $< E_{CM} <$ 150 Mev with any local or nonlocal potential, using momentum space equations, screened Coulomb and the renormalization approach.

pd $\rightarrow$ pd; pd $\rightarrow$ ppn; γ^3He $\leftrightarrow$ pd; γ^3He $\rightarrow$ ppn; e^{3}He $\longrightarrow$ e$'$pd; e^{3}He $\longrightarrow$ e$'$ppn

1.1 Technical Developments

- Use Alt, Grassberger and Sandhas equations (Faddeev like) for the transition operators;
- Use screened Coulomb

$$w_R(r) = w(r)\; e^{-(r/R)^n}$$

$$w(r) = \alpha_e/r \qquad \alpha \approx 1/137$$

$$n = 1 \Rightarrow \text{Yukawa Screening}$$

We use $4 \leq n \leq 6$

- Two-potential formula to separate long from short range contributions;
- Renormalization of the t-matrix (Taylor and Alt *et al.*)

1.2 Ex: pp Scattering

$$t^{(R)}(z) = (v + w_R) + (v + w_R) g_0(z) t^{(R)}(z)$$

$$t_R(z) = w_R + w_R\, g_0(z) t_R(z)$$

$$|\psi_R^+\rangle = [1 + g_0(z) t_R(z)]|\phi\rangle$$

One proves that the difference

$$[t^{(R)}(z) - t_R(z)] = [1 + t_R(z) g_0(z)] \tilde{t}^{(R)}(z) [1 + g_0(z) t_R(z)]$$

$$\tilde{t}^{(R)}(z) = v + v g_R(z) \tilde{t}^{(R)}(z)$$

$$g_R(z) = (z - h_0 - w_R)^{-1}$$

is a short range operator $\tilde{t}$ calculated between screened Coulomb waves. $\tilde{t}$ is well defined even in the $R \to \infty$ limit.

In the $R \to \infty$ limit, in each partial wave

$$z_R^{-\frac{1}{2}} \psi^+_{RL}(r) \approx \psi^+_{CL}(r)$$

$$z_R = e^{-i2(\sigma_L - \eta_{LR})}$$

It turns out that for sufficiently large R the renormalization becomes partial wave independent

$$z_R \xrightarrow[R\to\infty]{} e^{-i2\kappa[ln(2pR) - C/n]}$$

C = the Euler number,
n = power of the screening function,
$\kappa = \alpha_e \, \mu/p$,
p = on-shell momentum,
μ = pp reduced mass.

Semon and Taylor: Nuovo Cimento B23, 313 (1974); *ibid* A26, 48 (1975).

Therefore writing

$$t^{(R)}(z) = t_R(z) + [t^{(R)}(z) - t_R(z)]$$

and multiplying both sides by $z_R^{-\frac{1}{2}}$

$$\langle \vec{p}'\ \nu'|t|\ \vec{p}\ \nu\rangle = \lim_{R\to\infty}\{z_R^{-\frac{1}{2}}(p)\langle \vec{p}'\ \nu'|t^{(R)}(e+i0)|\ \vec{p}\ \nu\rangle z_R^{-\frac{1}{2}}(p)\}$$

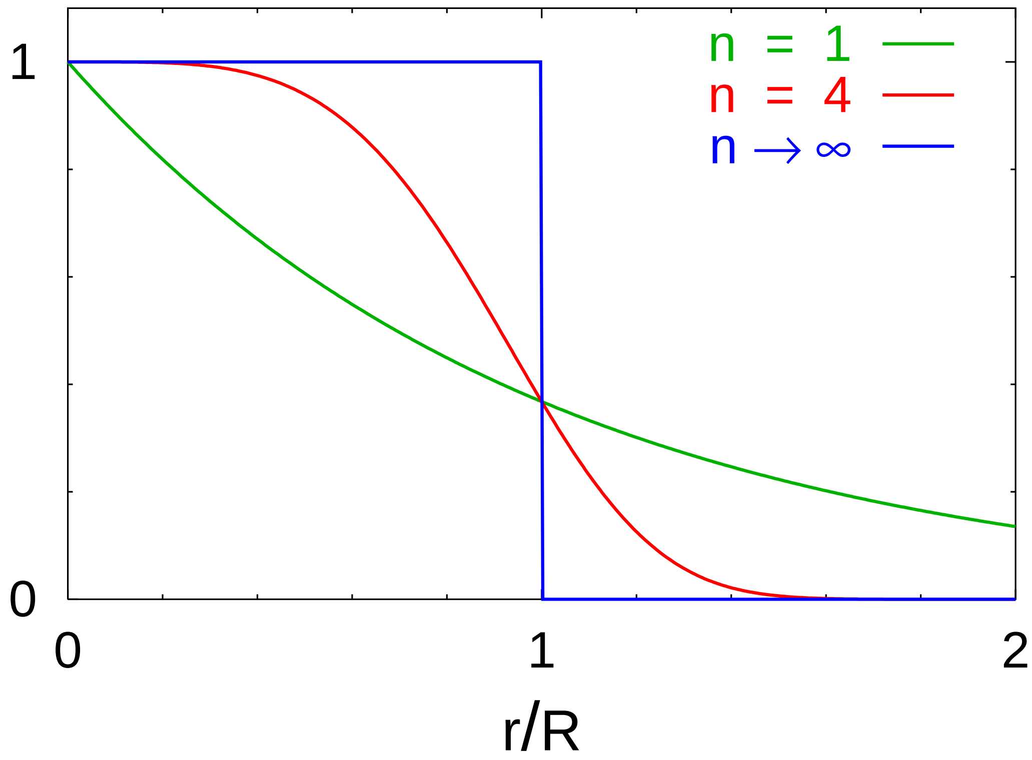
together with

$$\langle \vec{p}'\ \nu'|t_C|\ \vec{p}\ \nu\rangle = \lim_{R\to\infty}\{z_R^{-\frac{1}{2}}(p)\langle \vec{p}'\ \nu'|t_R(e+i0)|\ \vec{p}\ \nu\rangle z_R^{-\frac{1}{2}}(p)\}$$

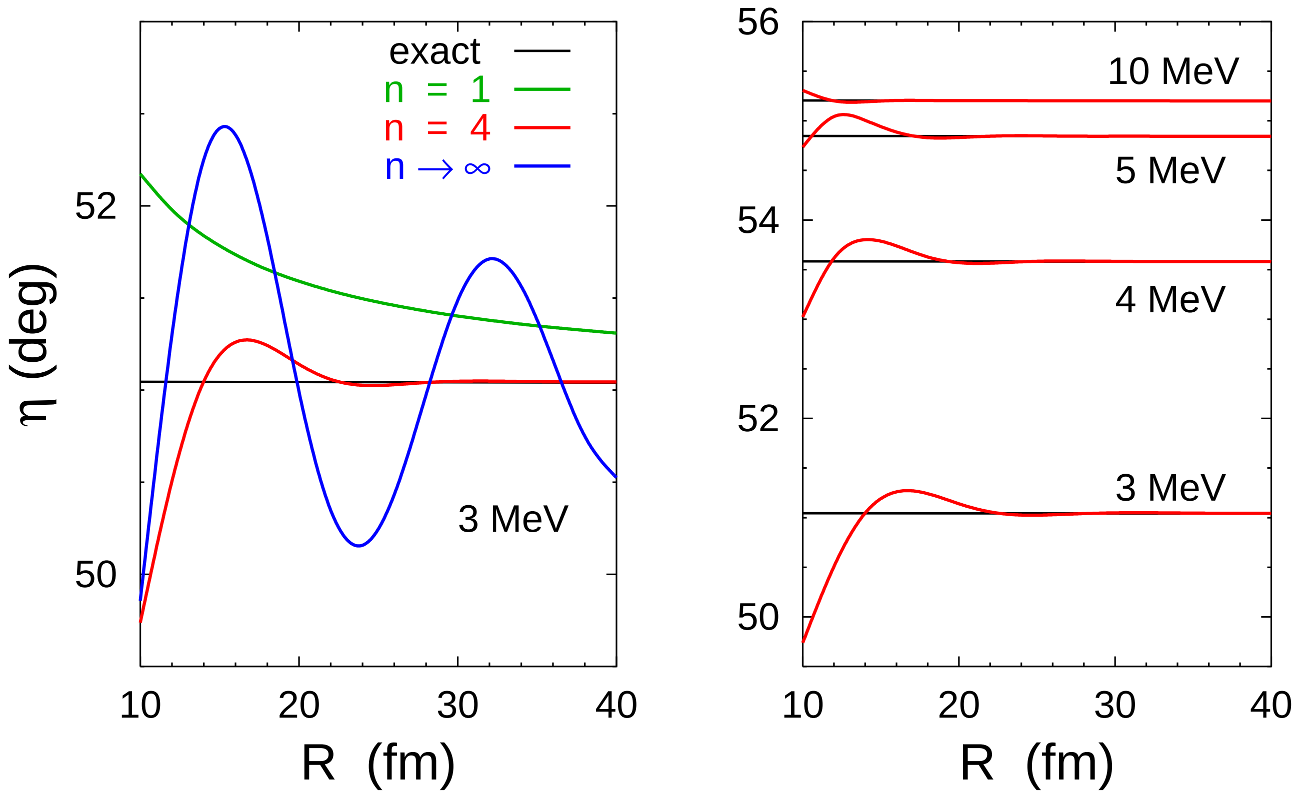
$$\langle \vec{p}'\ \nu'|t|\ \vec{p}\ \nu\rangle = \langle \vec{p}'\ \nu'|t_C|\ \vec{p}\ \nu\rangle + \lim_{R\to\infty}\{z_R^{-\frac{1}{2}}(p)\langle \vec{p}'\ \nu'|[t^{(R)}(e+i0) - t_R(e+i0)]|\ \vec{p}\ \nu\rangle z_R^{-\frac{1}{2}}(p)\}$$

Screened Coulomb potential

$$\frac{w_R(r)}{w(r)} = e^{-\left(\frac{r}{R}\right)^n}$$



pp scattering: $^1S_0$ phase shift



1.3 p+d Scattering

Three-body odd man out notation

$$\alpha = 1,\ 2\ \text{or}\ 3$$

$$v_\alpha = \text{hadronic pair interaction}$$

$$\omega_{\alpha R} = \text{screened Coulomb pair interaction}$$

If pair

$$\alpha = np \qquad \omega_{\alpha R} = 0$$

$$\alpha = pp \qquad \omega_{\alpha R} \neq 0$$

Let

$$U^{(R)}_{\beta\alpha}(Z) = \bar{\delta}_{\beta\alpha} G_0^{-1}(Z) + \sum_{\sigma} \bar{\delta}_{\beta\sigma} t^{(R)}_{\sigma}(Z) G_0(Z) U^{(R)}_{\sigma\alpha}(Z)$$

$$t^{(R)}_{\alpha}(Z) = (v_\alpha + w_{\alpha R}) + (v_\alpha + w_{\alpha R}) G_0(Z) t^{(R)}_{\alpha}(Z)$$

be the AGS equation for particle-pair scattering, and

$$U^{(R)}_{0\alpha}(Z) = G_0^{-1}(Z) + \sum_{\sigma} t^{(R)}_{\sigma}(Z) G_0(Z) U^{(R)}_{\sigma\alpha}(Z)$$

for breakup.

Let $W_{\alpha R}^{\text{c.m.}}$ be the screened Coulomb potential between one proton and the c.m. of the remaining pair, and $T_{\alpha R}^{\text{c.m.}}$ the corresponding t-matrix.

$W_{\alpha R}^{\text{c.m.}} = 0$ if α is a pp pair.

One may write

$$U_{\beta\alpha}^{(R)} = \underbrace{\delta_{\beta\alpha}\, T_{\alpha R}^{\text{c.m.}}}_{\text{long range}} + \underbrace{[U_{\beta\alpha}^{(R)} - \delta_{\beta\alpha}\, T_{\alpha R}^{\text{c.m.}}]}_{\text{short range}}$$

to subtract the singular part of $U_{\beta\alpha}^{(R)}$. Since the remainder can be identified with a short range operator we may proceed as before.

Using the renormalization prescription to reach the unscreened limit ($R \to \infty$):

$$\langle \phi_\beta(\vec{q}\,')\nu_\beta | U_{\beta\alpha} | \phi_\alpha(\vec{q})\nu_\alpha\rangle = \delta_{\beta\alpha}\langle \phi_\beta(\vec{q}\,')\nu_\beta | T^{\mathrm{c.m.}}_{\alpha C} | \phi_\alpha(\vec{q})\nu_\alpha\rangle$$

$$+ \lim_{R\to\infty}\{\mathcal{Z}^{-\frac{1}{2}}_{\beta R}(q')\langle \phi_\beta(\vec{q}\,')\nu_\beta | [U^{(R)}_{\beta\alpha}(E_\alpha + i0) - \delta_{\beta\alpha} T^{\mathrm{c.m.}}_{\alpha R}(E_\alpha + i0)]$$

$$\times | \phi_\alpha(\vec{q})\nu_\alpha\rangle \mathcal{Z}^{-\frac{1}{2}}_{\alpha R}(q)\}$$

For $\mathcal{Z}_{\alpha R}$ we use the partial wave dependent form that leads to faster convergence.

and

$$\langle\phi_0(\vec{p}'\,\vec{q}')\nu_0|U_{0\alpha}|\phi_\alpha(\vec{q})\nu_\alpha\rangle = \lim_{R\to\infty}\{z_R^{-\frac{1}{2}}(p')\langle\phi_0(\vec{p}'\,\vec{q}')\nu_0|$$

$$\times U_{0\alpha}^{(R)}(E_\alpha + i0)|\phi_\alpha(\vec{q})\nu_\alpha\rangle\mathcal{Z}_{\alpha R}^{-\frac{1}{2}}(q)\}$$

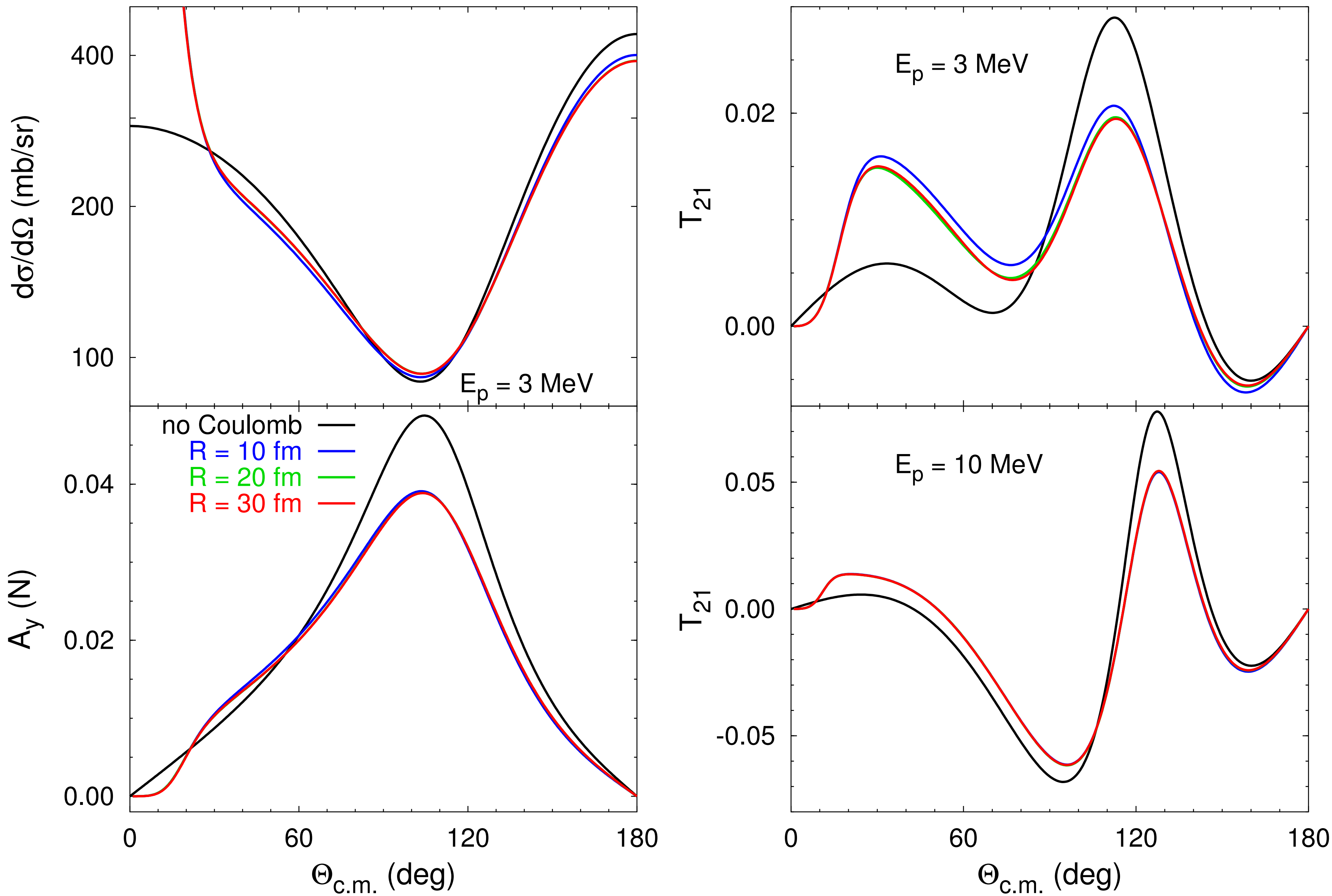
$$\mathcal{Z}_{\alpha R}(q) = e^{-2i\kappa_\alpha(q)[\ln(2qR)-C/n]},$$

$$z_R(p) = e^{-2i\kappa(p)[\ln(2pR)-C/n]},$$

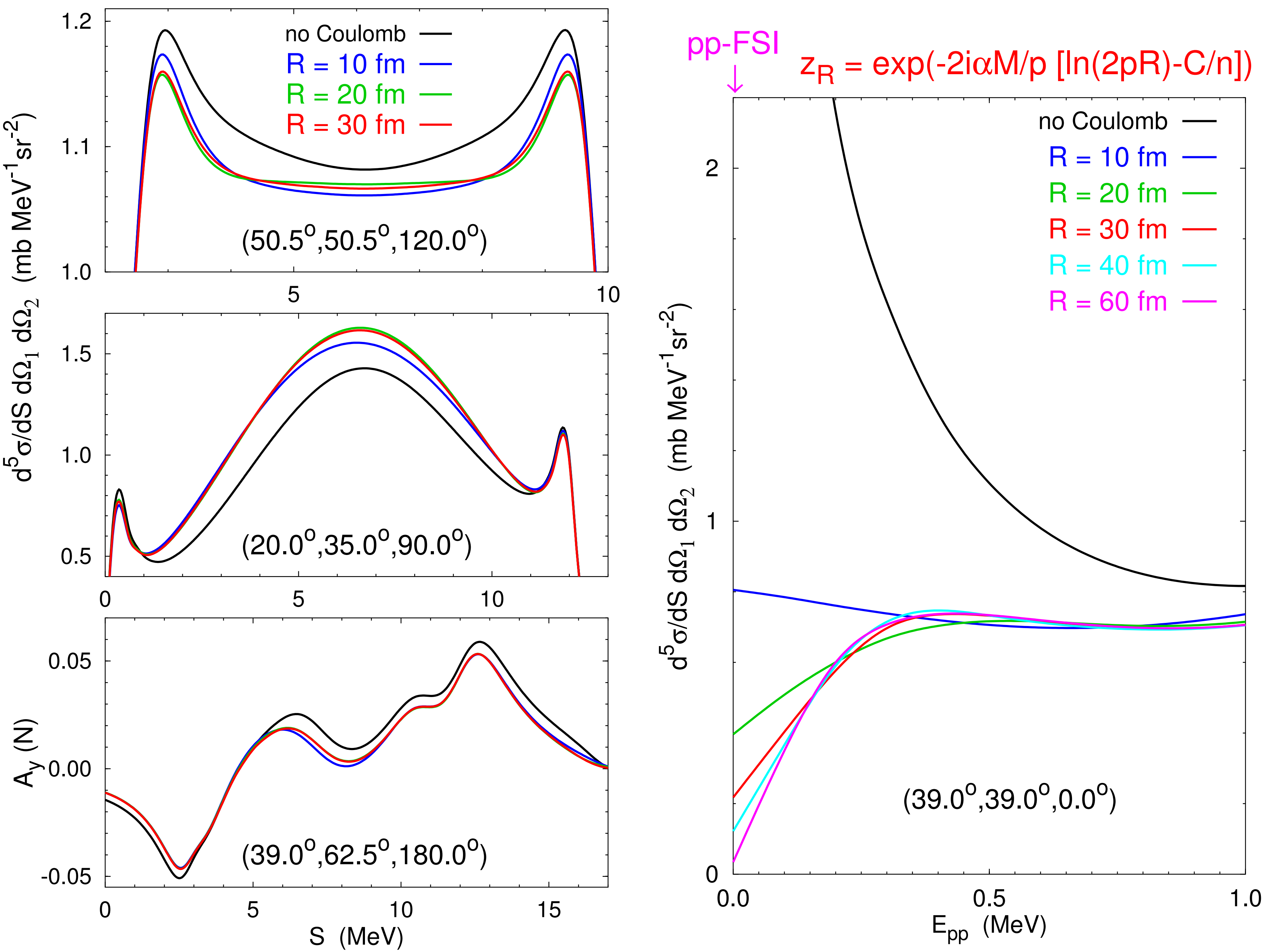
$\kappa_\alpha(q) = \alpha_e\, M_\alpha/q$ **pd Coulomb parameter**

$\kappa(p) = \alpha_e\, \mu/p$ **pp Coulomb parameter**

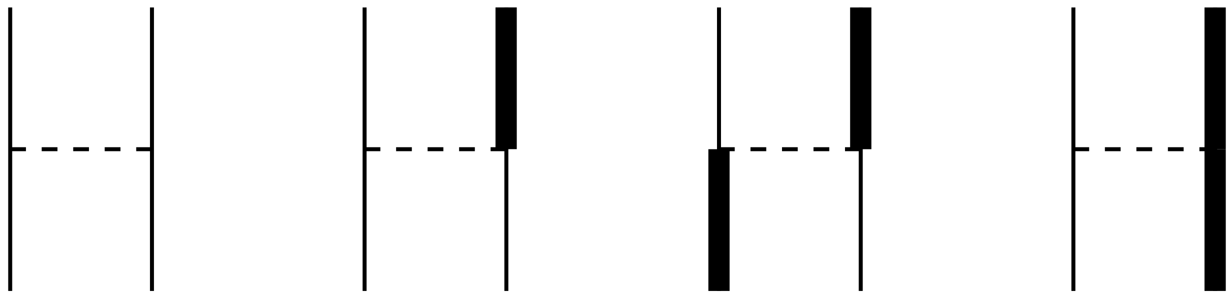
Convergence with *R*: *pd* elastic scattering



Convergence with R: pd breakup at $E_p = 13$ MeV



Two-baryon coupled-channel potential



Exchange of π, ρ, ω and σ mesons

CD Bonn + Δ: $\chi^2/\text{datum} = 1.02$

Beneficial for properties of 3N bound state: $E_B = -8.30\,\text{MeV}$

Results

CD Bonn + Δ

CD Bonn + Δ + Coulomb

CD Bonn + Coulomb

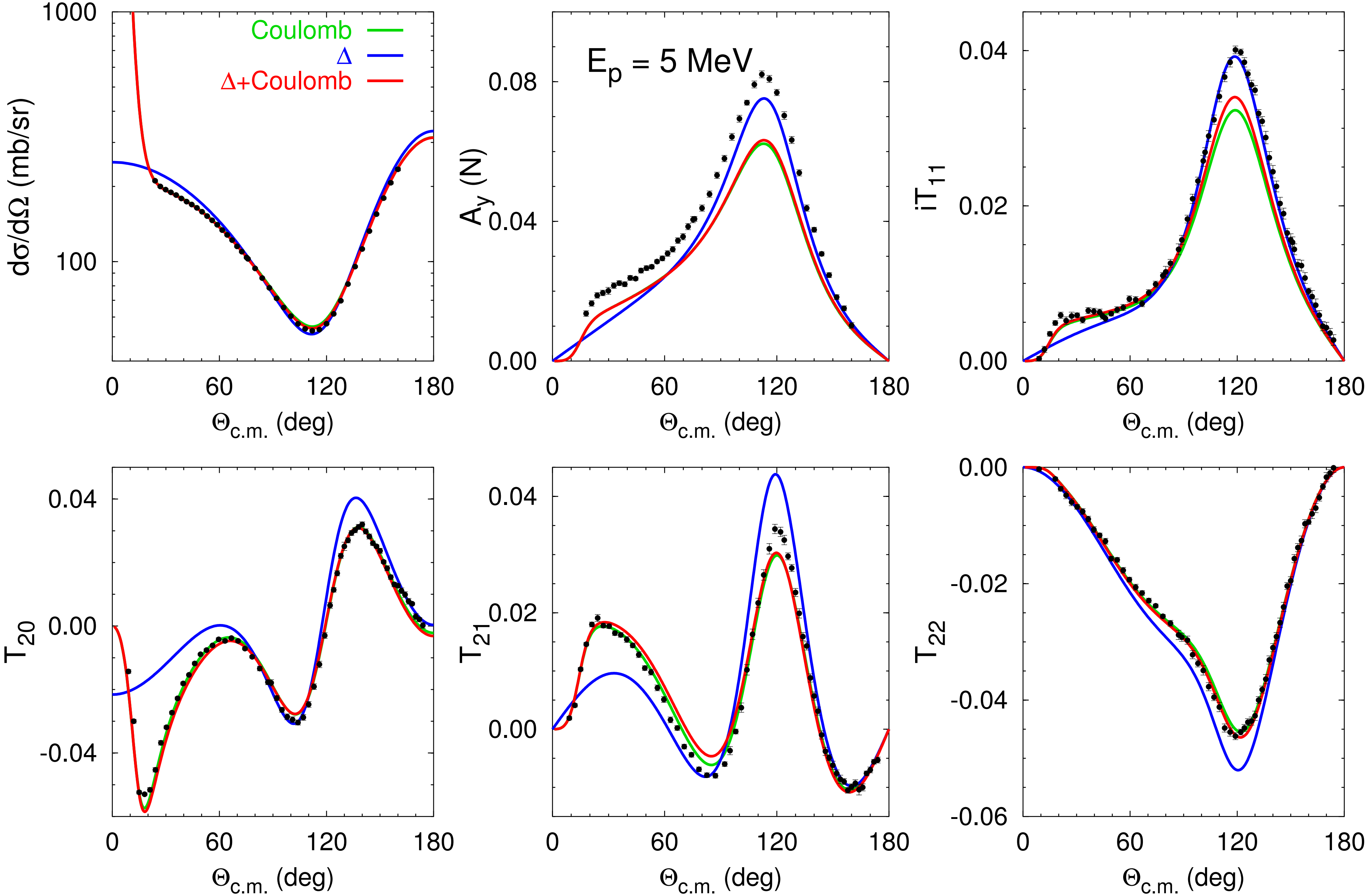
} Coulomb effect

} Δ effect

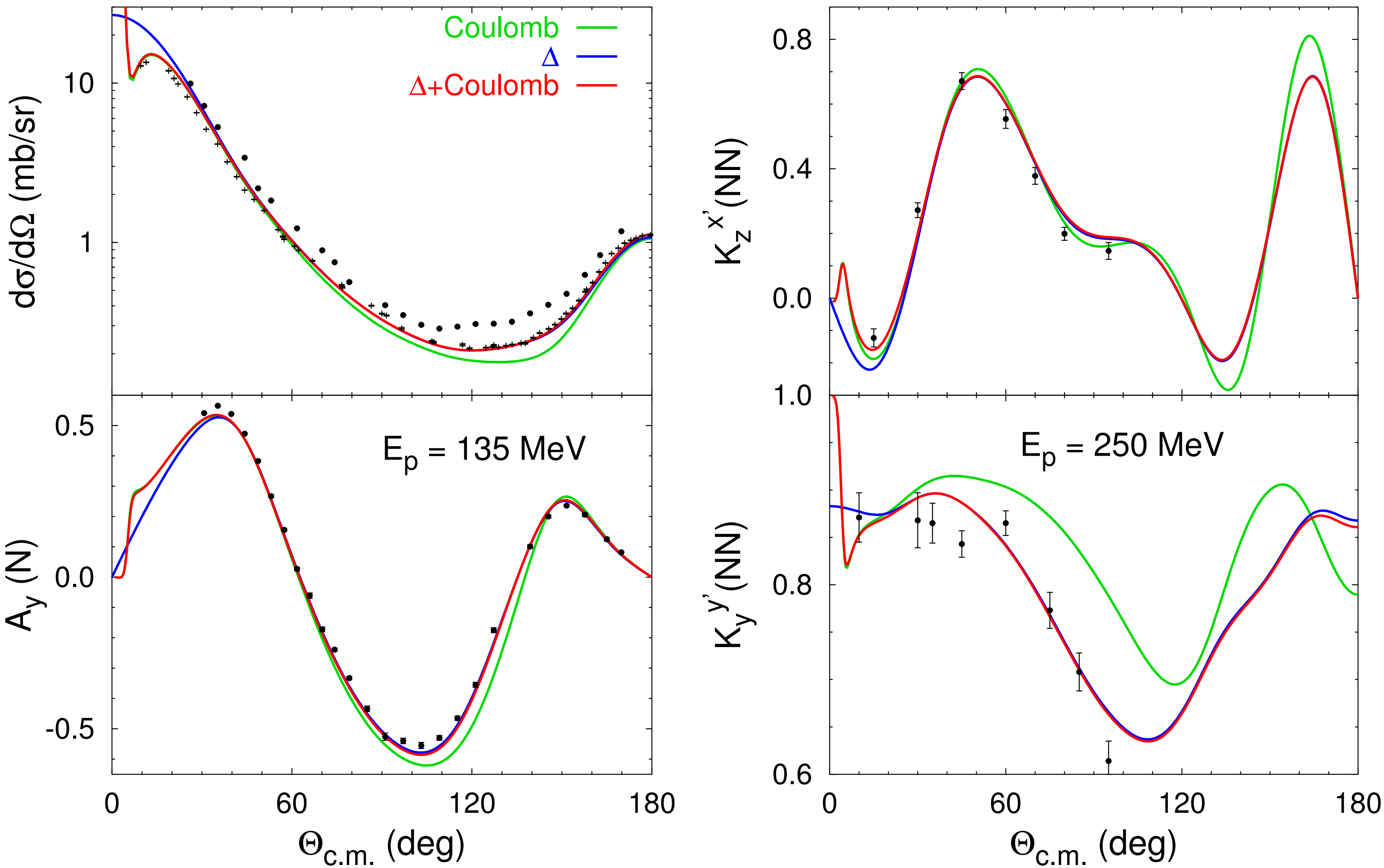
Δ isobar:

- effective 3NF
  - ▷ Fujita-Miyazawa, Illinois, ...
  - ▷ π, ρ, ω, σ exchanges
- effective 2N and 3N currents

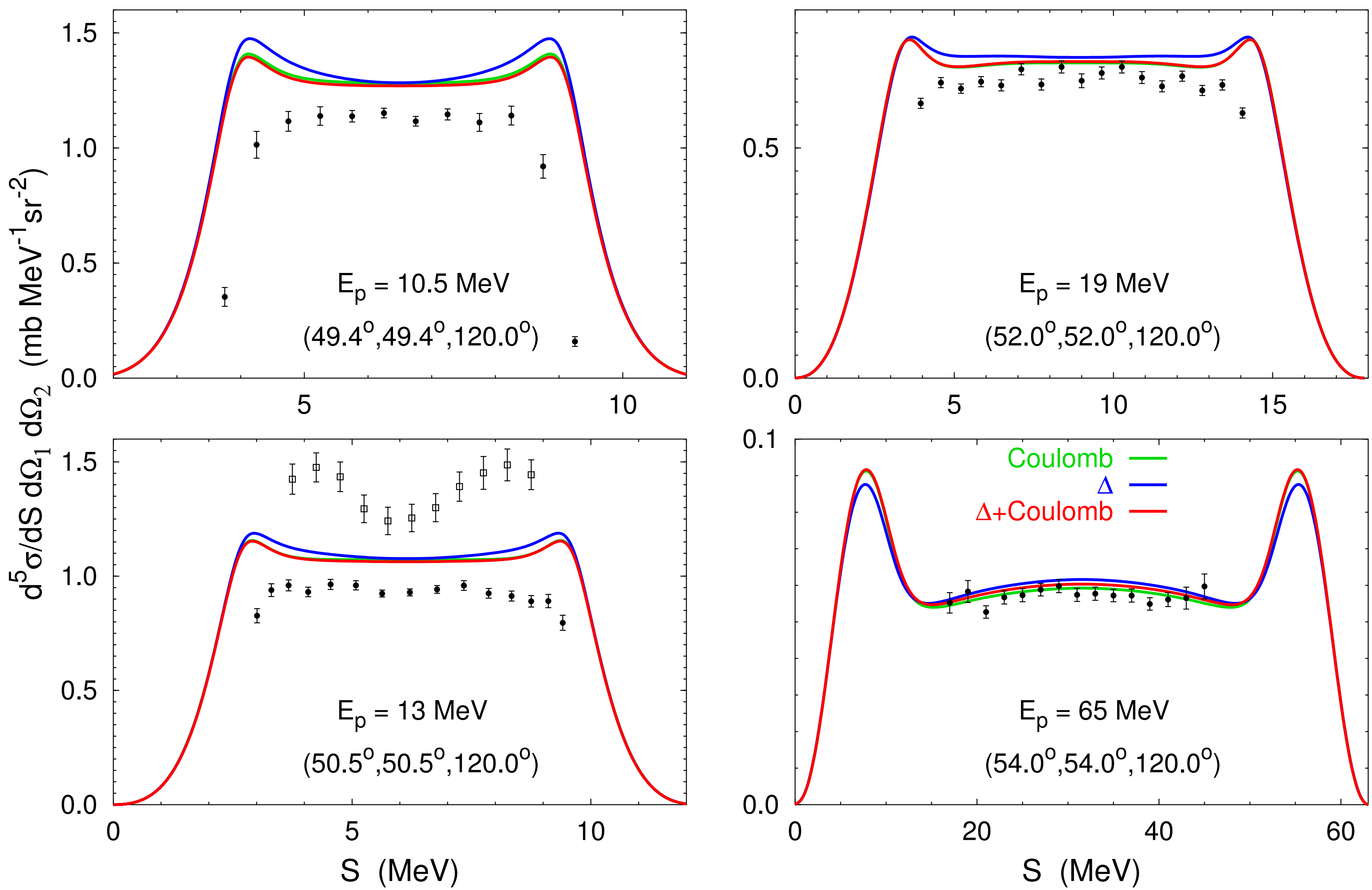
pd elastic scattering at low energies



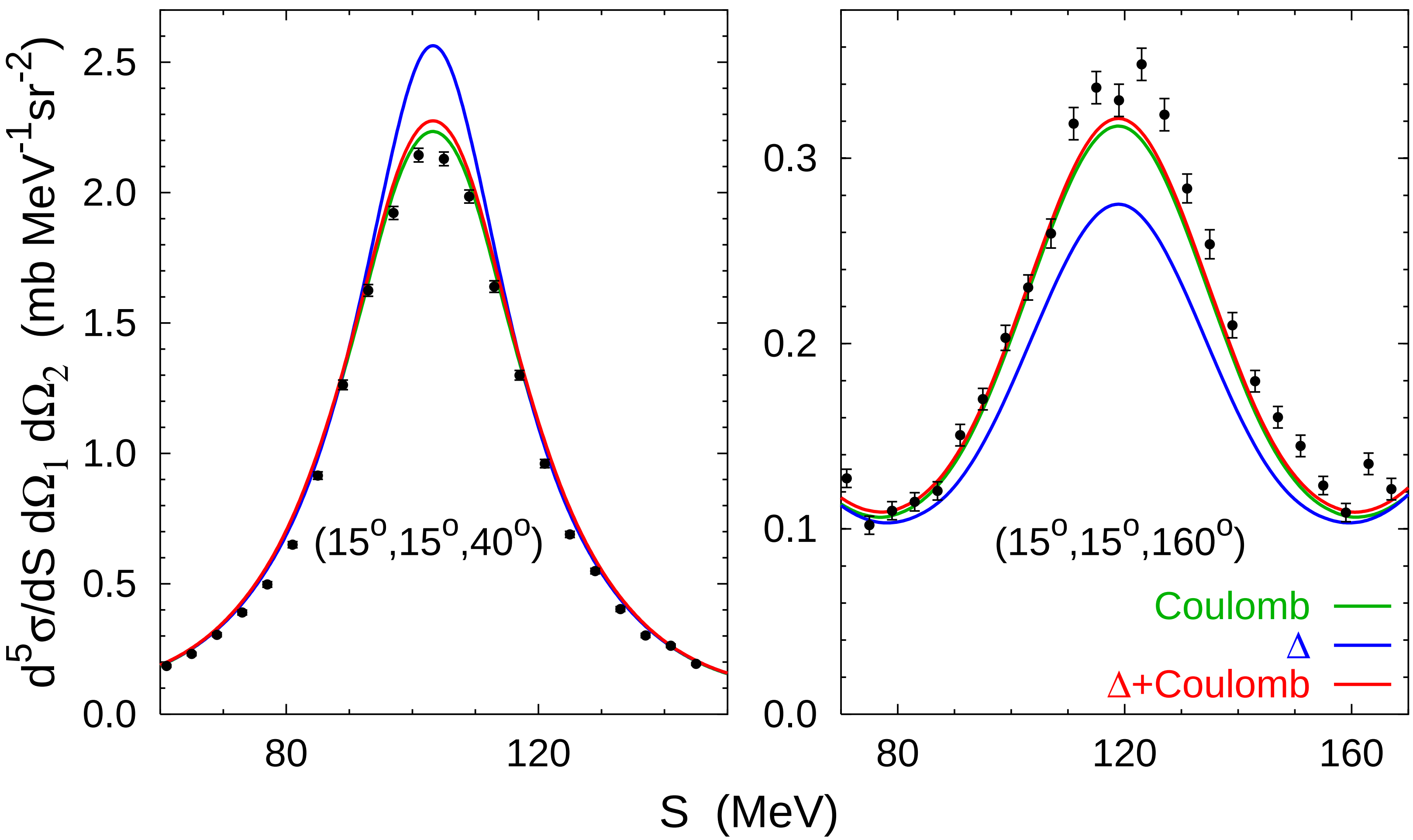
pd elastic scattering at higher energies



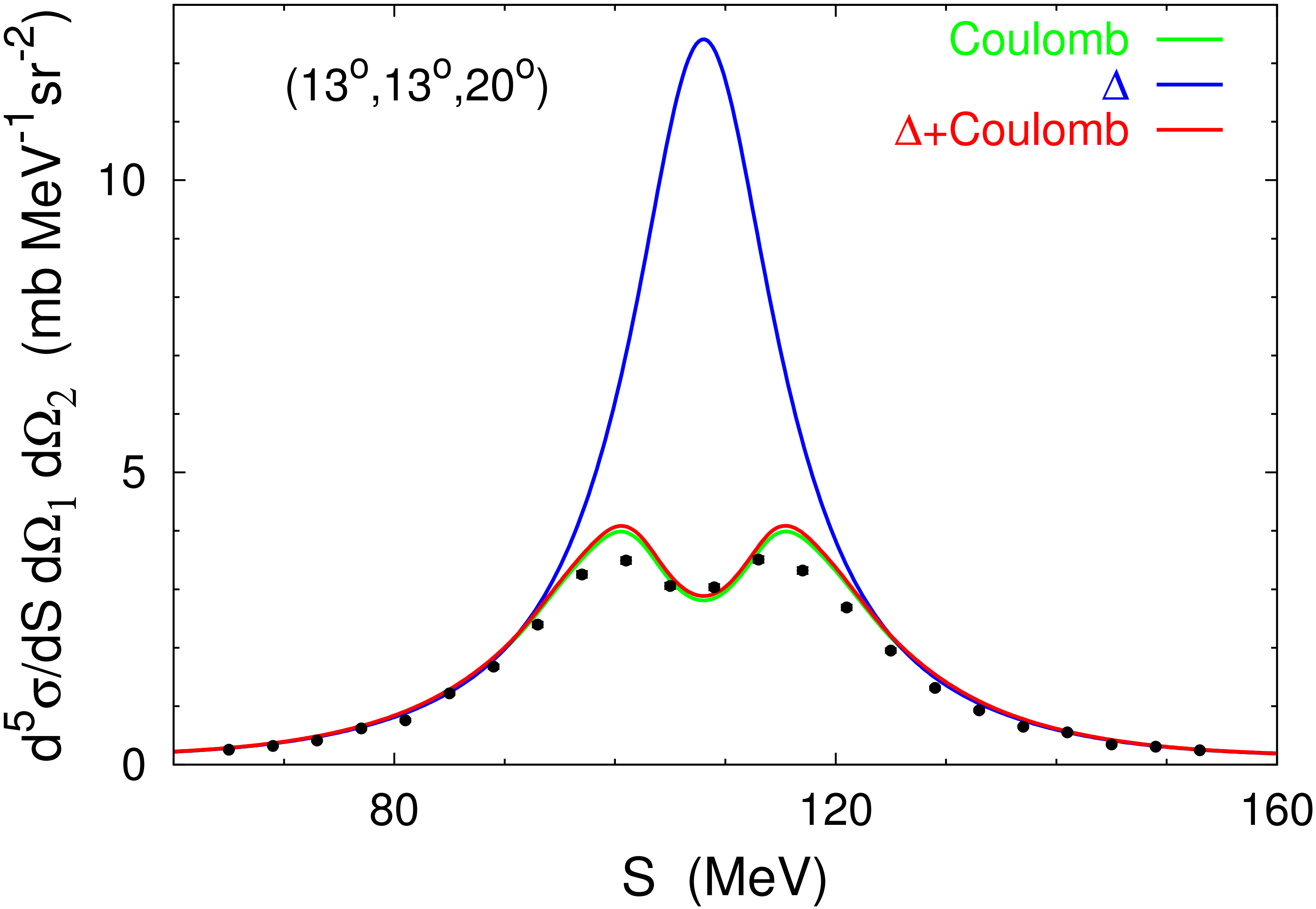
pd breakup: space-star configurations



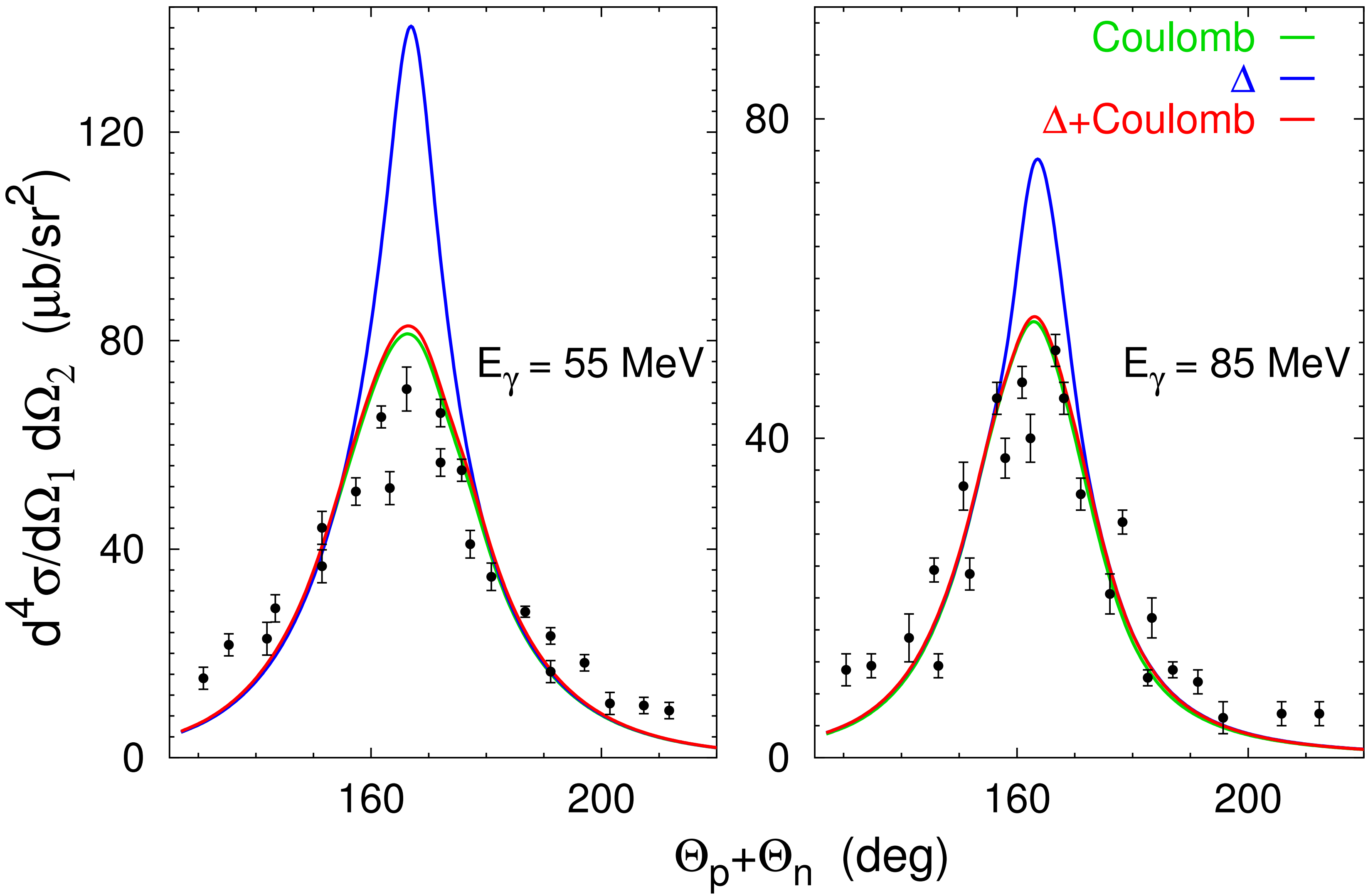
dp breakup at $E_d = 130$ MeV



dp breakup at $E_d = 130$ MeV close to pp-FSI



Three-nucleon photodisintegration $^3\mathrm{He}(\gamma, pn)p$



2. FOUR-NUCLEON REACTIONS

The 4N scattering problem gives rise to the simplest set of nuclear reactions that shows the complexity of heavier systems

$$\left\{\begin{array}{l} n\,^3\mathrm{H} \to n\,^3\mathrm{H} \\ \\ p\,^3\mathrm{He} \to p\,^3\mathrm{He} \end{array}\right. \qquad \text{dominated by isospin } \mathcal{T} = 1$$

$$\left\{\begin{array}{ll} dd \to dd & \text{dominated by isospin } \mathcal{T} = 0 \\ \quad\;\; \to n\,^3\mathrm{He} & \multirow{2}{*}{} \\ \quad\;\; \to p\,^3\mathrm{H} & \end{array}\right. \quad \left.\begin{array}{l} \to n\,^3\mathrm{He} \\ \to p\,^3\mathrm{H} \end{array}\right\} \mathcal{T} = 0 + \mathcal{T} = 1$$

$$\left\{\begin{array}{l} n\,^3\mathrm{He} \to n\,^3\mathrm{He} \\ \qquad\quad\; \to p\,^3\mathrm{H} \quad \text{mixed isospin } \mathcal{T} = 0 \text{ and } \mathcal{T} = 1 \\ \qquad\quad\; \to dd \end{array}\right.$$

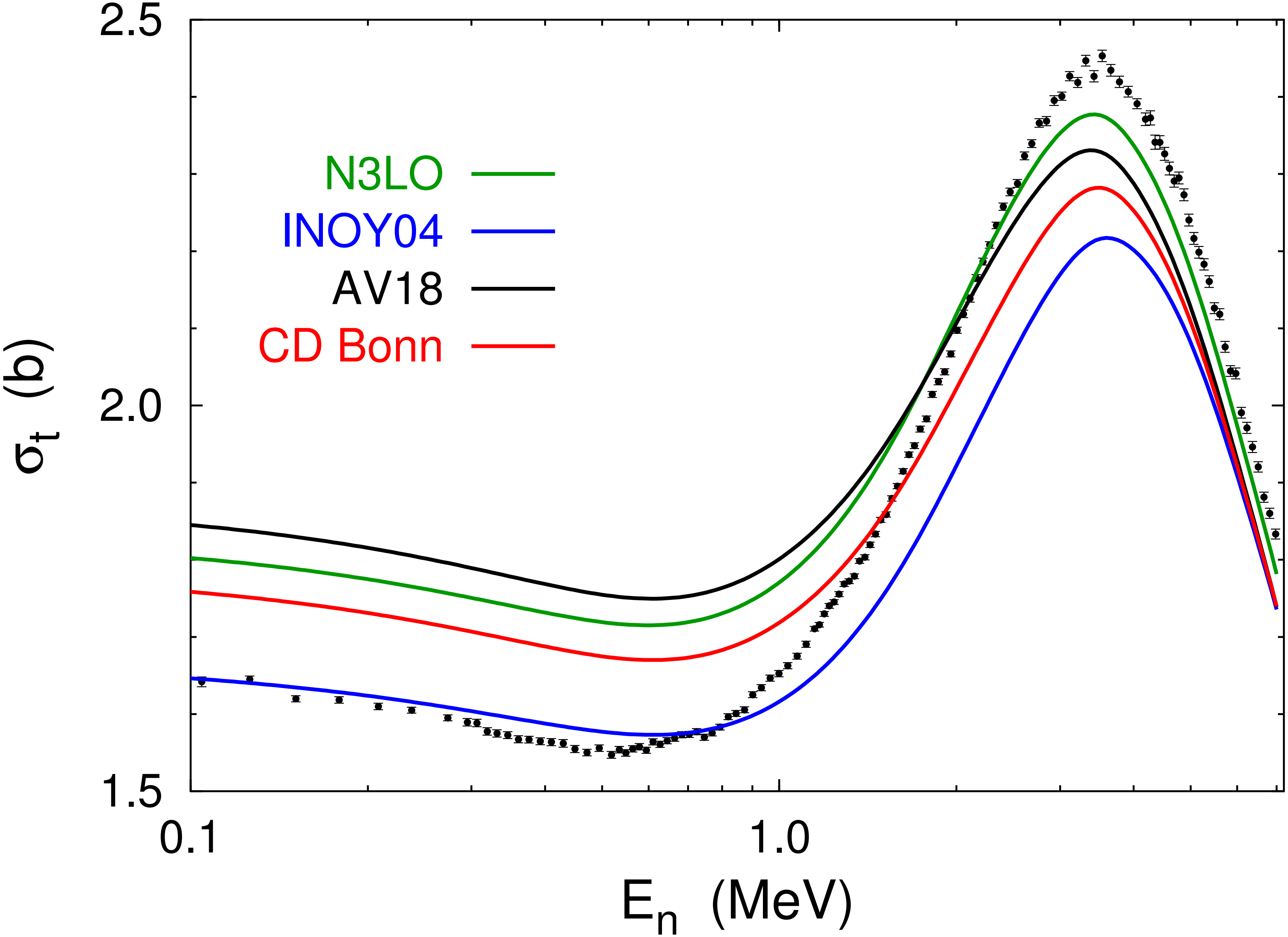
Parallel talk by *Arnas Deltuva*

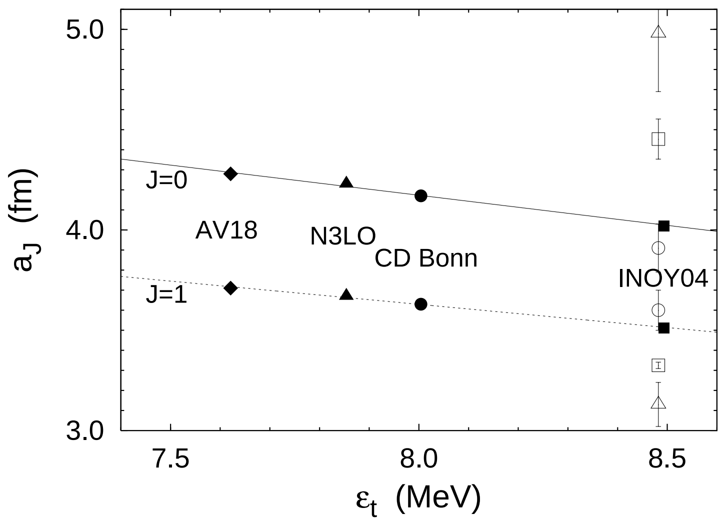
2.1 “Complexity Digest”

These are three-variable integral equations:

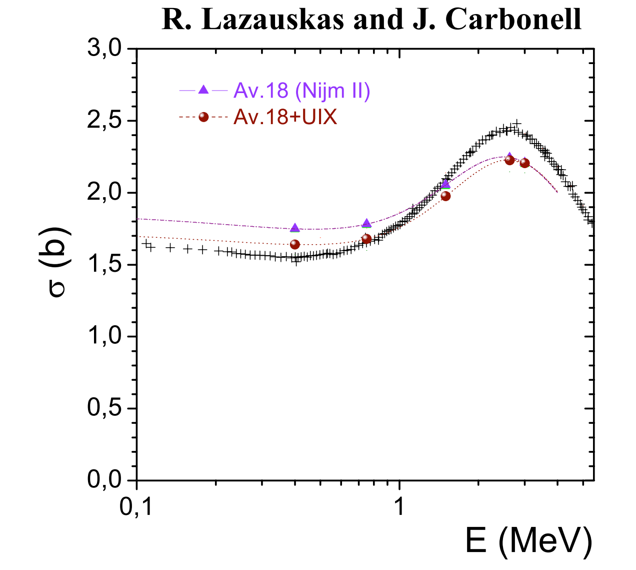
- **Triple partial wave expansion;**
- **Triple discretization of Jacobi momenta;**
- **Gaussian integration;**
- **Spline interpolation;**
- **Include up to 15000 partial waves (combined 2N, 3N, 4N);**
- **System of $> 10^8$ linear equations (size of the kernel $\approx 10^8$ GB);**
- **Summing up the Neumann series by Padé method.**

n-^{3}H total cross section

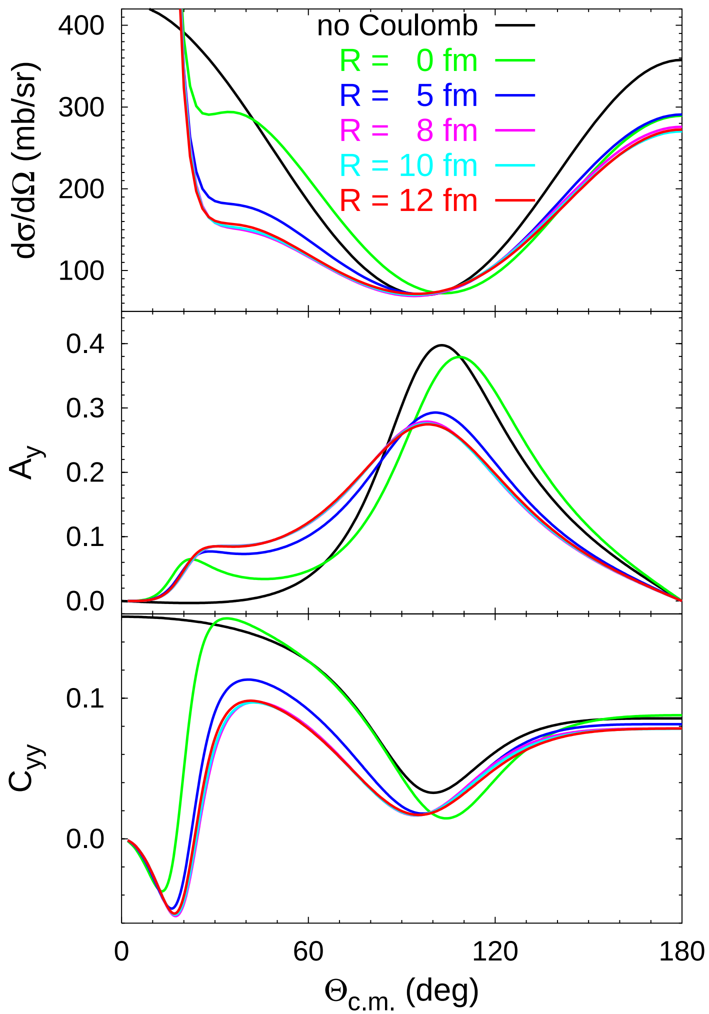




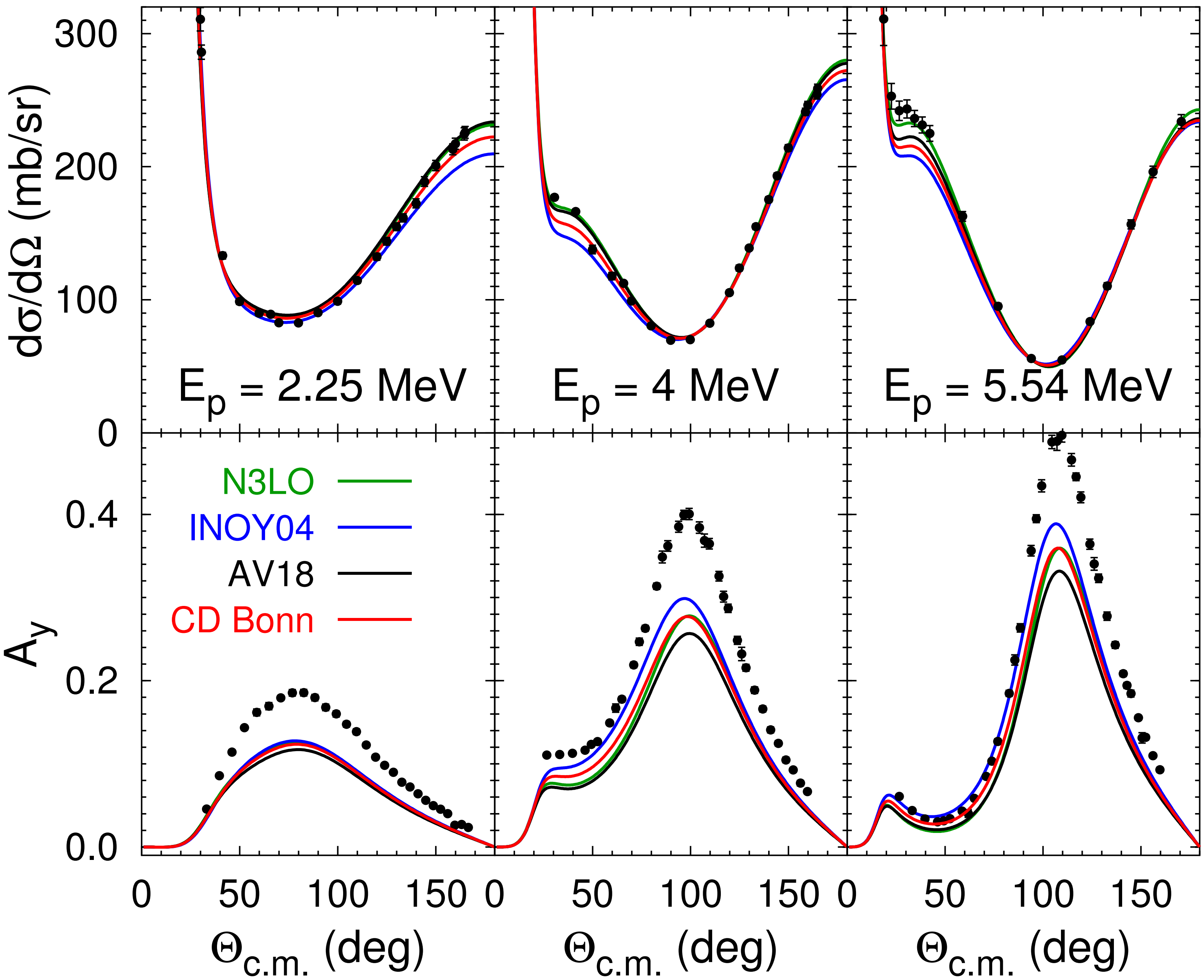
	ε_t	ε_α	a_0	a_1	σ_t (0)	σ_t (3.5)
AV18	**7.621**	**24.24**	**4.28**	**3.71**	**1.88**	**2.33**
Nijmegen II	**7.653**	**24.50**	**4.27**	**3.71**	**1.87**	**2.31**
Nijmegen I	**7.734**	**24.94**	**4.25**	**3.69**	**1.85**	**2.30**
N3LO	**7.854**	**25.38**	**4.23**	**3.67**	**1.83**	**2.38**
CD Bonn	**7.998**	**26.11**	**4.17**	**3.63**	**1.79**	**2.28**
INOY04	**8.493**	**29.11**	**4.02**	**3.51**	**1.67**	**2.22**


R. Lazauskas and J. Carbonell
Av.18 (Nijm II)
Av.18+UIX
σ (b)
E (MeV)
3,0
2,5
2,0
1,5
1,0
0,5
0,0
0,1
1

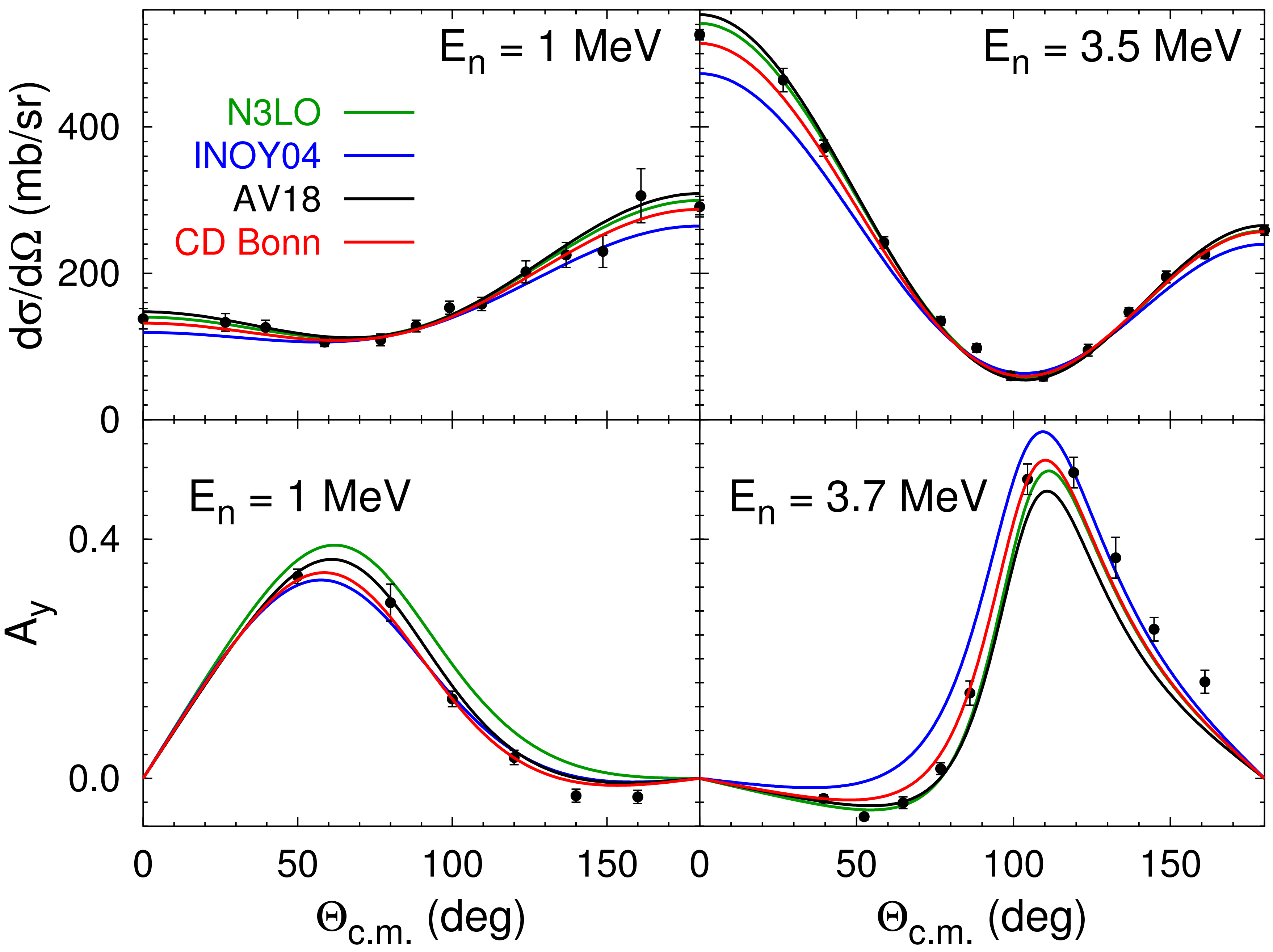
p-^{3}He OBSERVABLES at $E_p = 4$ MeV

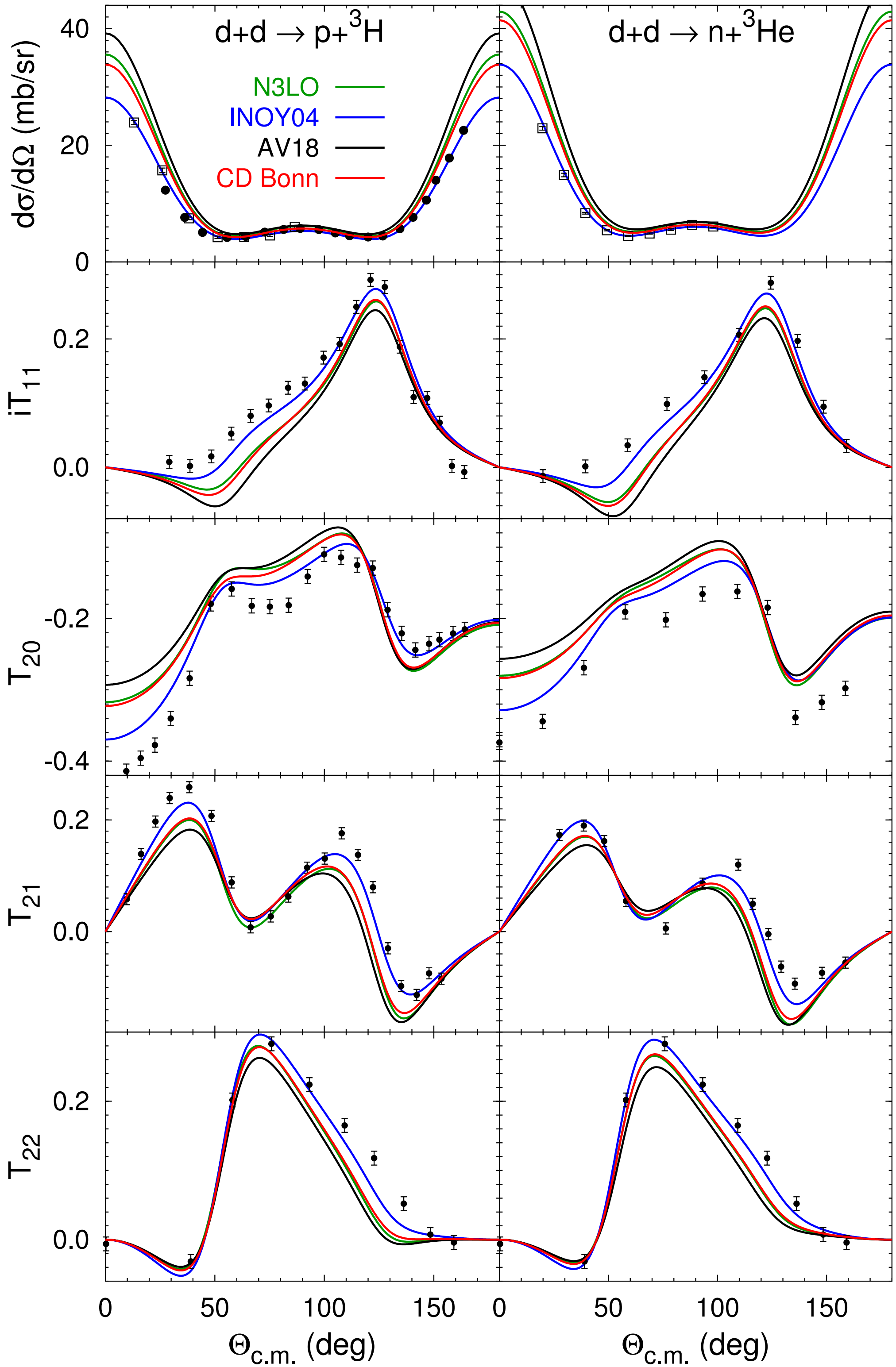


p-^{3}He scattering



n-^{3}He scattering





$d + d \rightarrow p + {}^3\mathrm{H}$

and

$d + d \rightarrow n + {}^3\mathrm{He}$

transfer

3. THREE-BODY APPROACH TO DIRECT NUCLEAR REACTIONS

Given the recent success of including Coulomb in exact three-body calculations, the goal is:

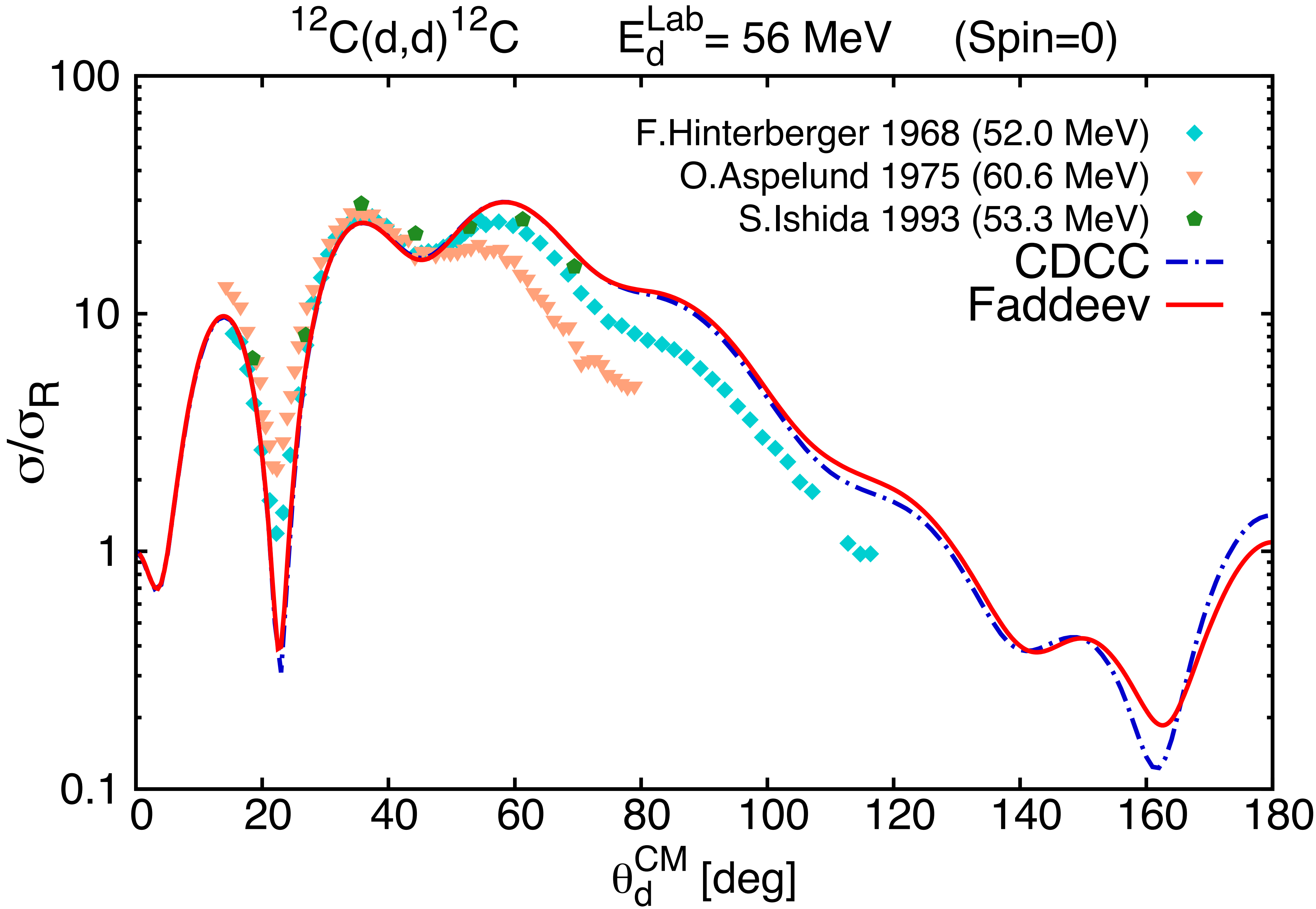
- **To explore the application of Faddeev/AGS equations to direct nuclear reactions driven by deuterons or halo nuclei;**
- **Compare with equivalent CDCC calculations;**
- **Provide a more reliable description of the data;**
- **Hopefully being able to extract better structure information such as spectroscopic factor, rms radius etc.**

3.1 Model Problems

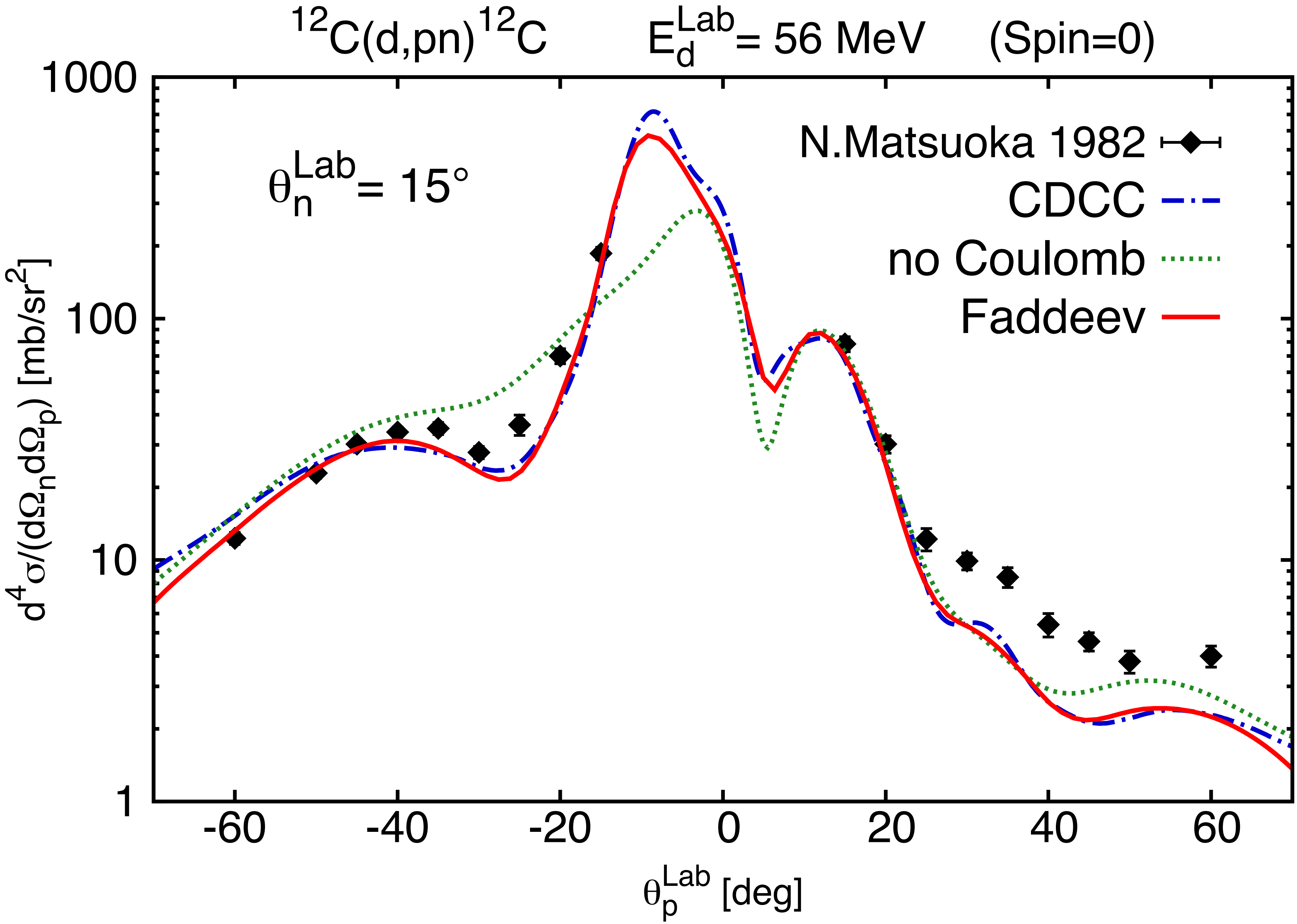
How reliable is CDCC compared to exact solutions for the same effective three-body problem.

	^{11}Be + p scattering		d + ^{12}C scattering
	$(n + {}^{10}\text{Be} + p)$		$(n + p + {}^{12}\text{C})$
$p - {}^{10}\text{Be}$:	optical potential plus Coulomb	$n - {}^{12}\text{C}$:	optical potential at $\frac{1}{2}$ energy
$n - {}^{10}\text{Be}$:	real potential that supports an s - wave bound state p - wave excited state d - wave resonance	$p - {}^{12}\text{C}$:	optical potential at $\frac{1}{2}$ energy plus Coulomb
$n - p$:	real potential that supports a bound state at the deuteron binding energy	$n - p$:	real potential that supports a bound state at the deuteron binding energy

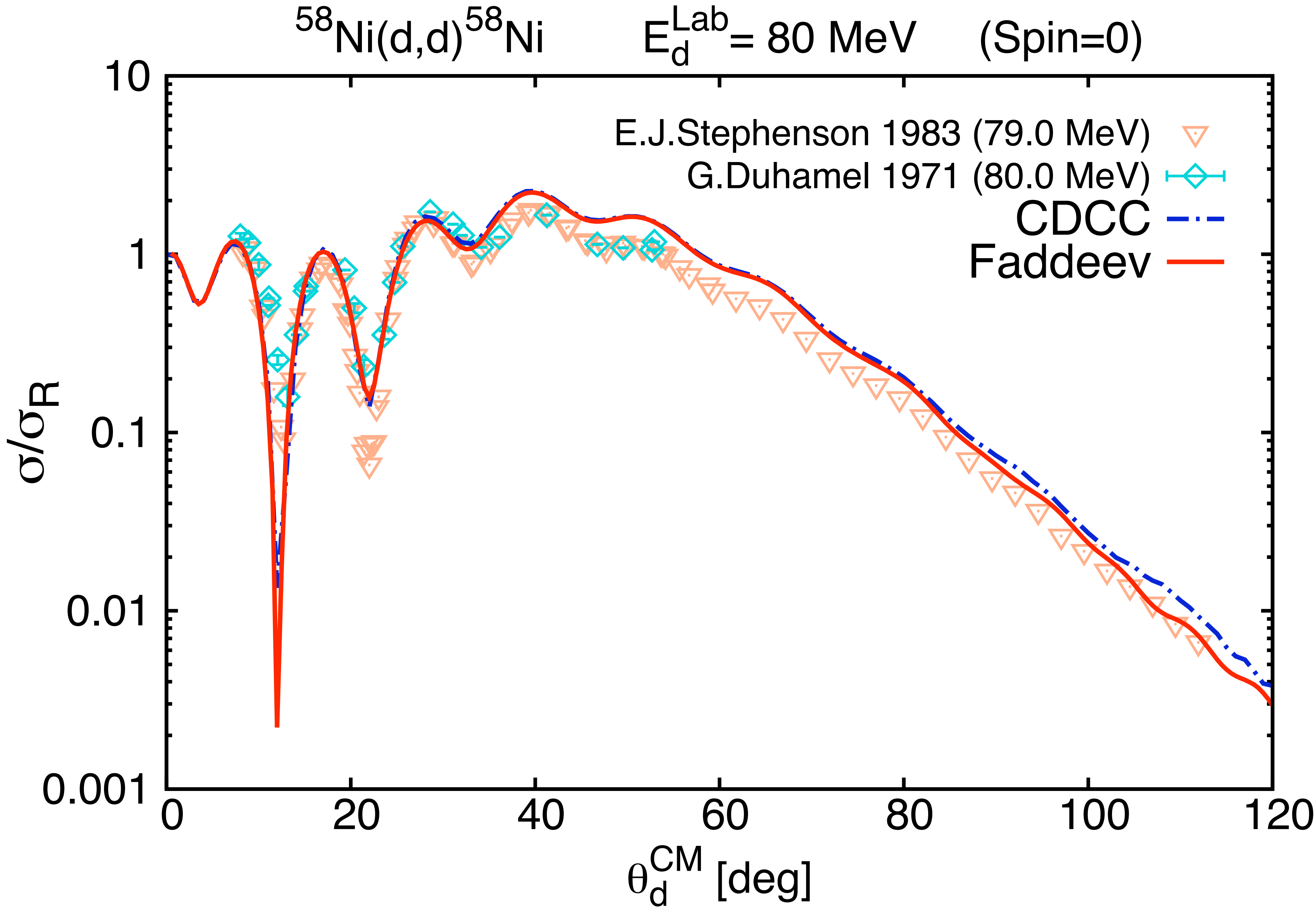
See Poster



$^{12}C(d,d)^{12}C$ E_d^{Lab}= 56 MeV (Spin=0)
F.Hinterberger 1968 (52.0 MeV)
O.Aspelund 1975 (60.6 MeV)
S.Ishida 1993 (53.3 MeV)
CDCC
Faddeev
σ/σ_R
100
10
1
0.1
0
20
40
60
80
100
120
140
160
180
θ_d^{CM} [deg]



$^{12}C(d,pn)^{12}C$ E_d^{Lab}= 56 MeV (Spin=0)
θ_n^{Lab}= 15°
N.Matsuoka 1982
CDCC
no Coulomb
Faddeev
$d^4\sigma/(d\Omega_n d\Omega_p)$ [mb/sr^2]
θ_p^{Lab} [deg]
1000
100
10
1
-60
-40
-20
0
20
40
60



^{58}Ni(d,d)^{58}Ni
E_d^{Lab}= 80 MeV
(Spin=0)
E.J.Stephenson 1983 (79.0 MeV)
G.Duhamel 1971 (80.0 MeV)
CDCC
Faddeev
10
1
0.1
0.01
0.001
0
20
40
60
80
100
120
σ/σ_R
θ_d^{CM} [deg]

3.2 CDCC

1 - CDCC-BU

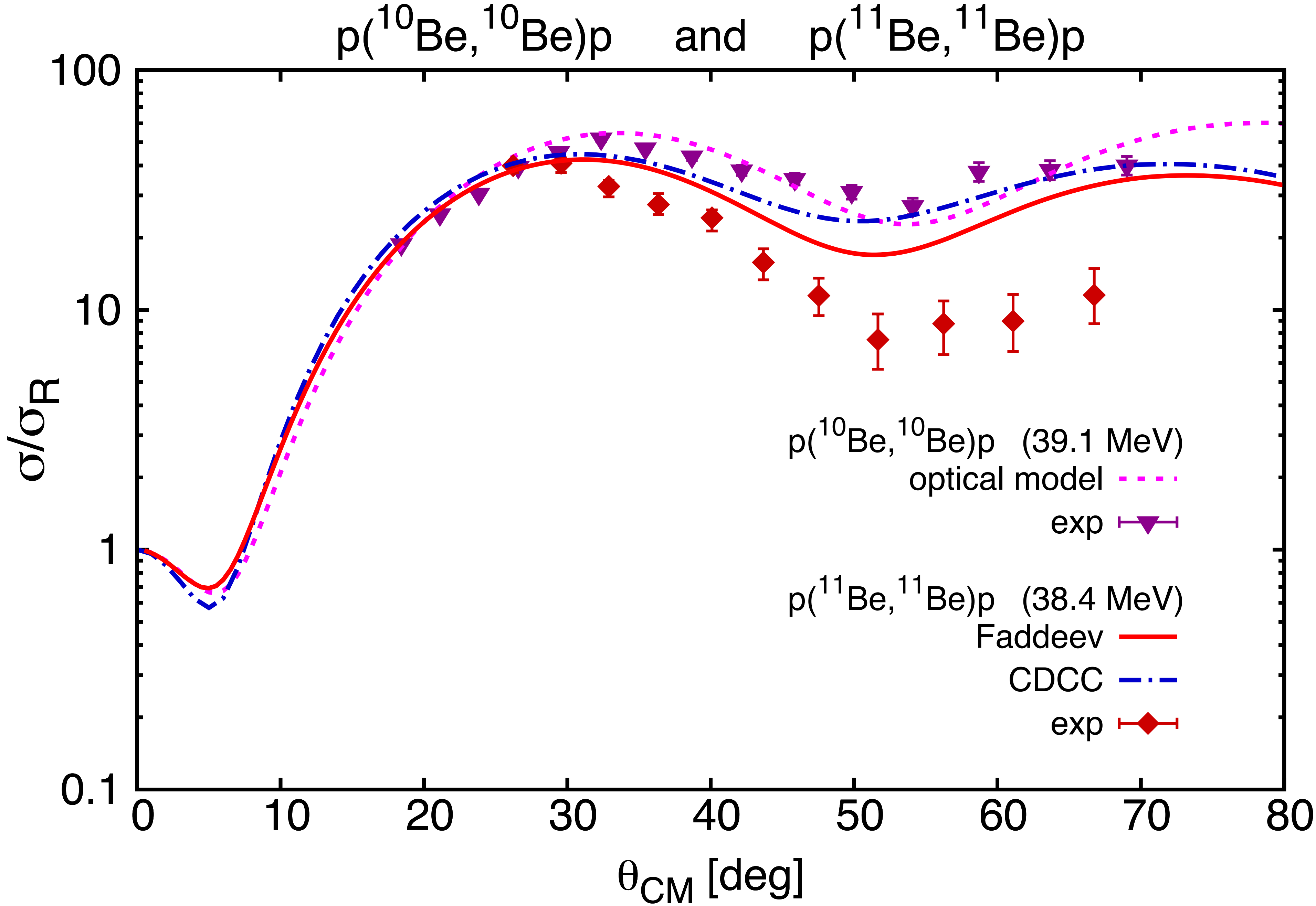
The wave function is expressed in terms of the continuum states of $^{10}\mathrm{Be} + n$.

This is direct breakup (BU) where $^{11}\mathrm{Be} + p \longrightarrow (^{10}\mathrm{Be} + n) + p$.

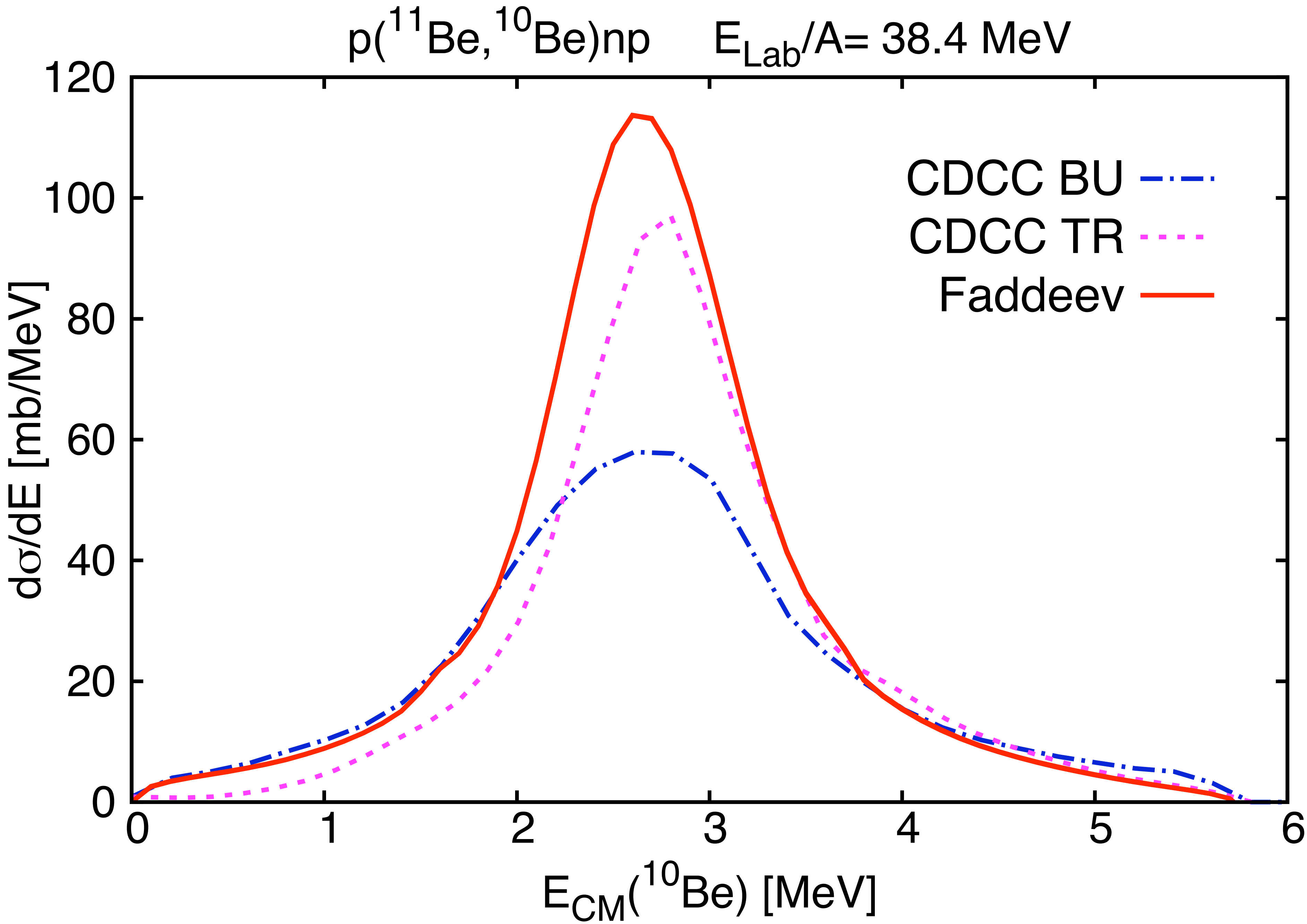
2 - CDCC-TR

The wave function is expressed in terms of the continuum states of $n - p$.
This is the transfer of the neutron to the continuum of the deuteron (TR) $^{11}\mathrm{Be} + p \longrightarrow ^{10}\mathrm{Be} + (n + p)$.

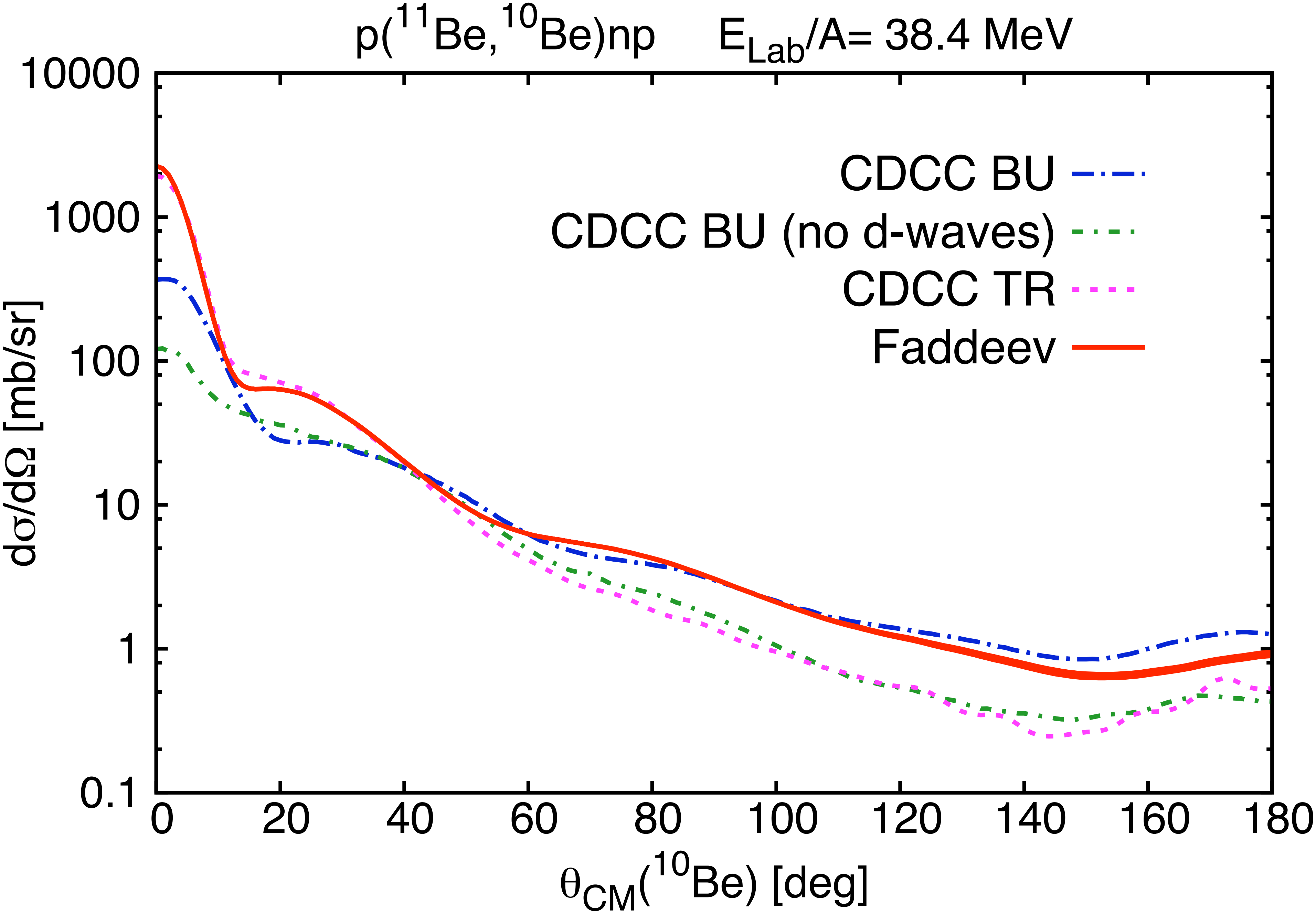
In Nucl. Phys. A767, 138 (2006) A. Moro and F. Nunes showed that breakup observables for both processes do not coincide.



p(10Be,10Be)p and p(11Be,11Be)p
σ/σR
θCM [deg]
100
10
1
0.1
0
10
20
30
40
50
60
70
80
p(10Be,10Be)p (39.1 MeV)
optical model
exp
p(11Be,11Be)p (38.4 MeV)
Faddeev
CDCC
exp



p(^{11}Be,^{10}Be)np E$_{Lab}$/A= 38.4 MeV
CDCC BU
CDCC TR
Faddeev
dσ/dE [mb/MeV]
E$_{CM}$(^{10}Be) [MeV]
0
20
40
60
80
100
120
0
1
2
3
4
5
6



p(11Be,10Be)np E_Lab/A= 38.4 MeV
CDCC BU
CDCC BU (no d-waves)
CDCC TR
Faddeev
dσ/dΩ [mb/sr]
θ_CM(10Be) [deg]
10000
1000
100
10
1
0.1
0
20
40
60
80
100
120
140
160
180

CONCLUSIONS

– Coulomb effects in three-body calculations can now be reliably included both in elastic scattering and breakup.

– In breakup reactions Coulomb effects can be very large depending on the relative $p-p$ momentum in the final state.

– *Ab initio* 4N calculations are now as reliable and accurate as 3N calculations, and Coulomb may be included.

– Presently known NN force models badly fail to reproduce σ_t in $n-{}^3$H scattering, and 3N forces may not bring a cure. Further investigations are needed.

– We have a 4N A_y problem (as in 3N) but it is much larger in $p-{}^3$He than in $n-{}^3$He.

- **Reactions driven by $d - d$ look surprisingly good, particularly if the NN interaction reproduces ^{3}H and ^{3}He binding.**

- **Faddeev/AGS calculations can be successfully applied to direct nuclear reactions, where Coulomb effects are larger.**

- **CDCC calculations fail badly in $p + {}^{11}$Be elastic scattering and breakup but do well in $d - {}^{12}$C.**