Shell-model studies on exotic nuclei around $^{132}$Sn

Luigi Coraggio

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The $^{132}$Sn region

- Neutron-rich nuclei: an interesting laboratory to test the role of the residual $NN$ interaction
The $^{132}$Sn region

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- Approaching the neutron drip-line: is there some new physics coming out?
  Radioactive ion beams facilities: more data in the pipeline
The $^{132}$Sn region

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- Approaching the neutron drip-line: is there some new physics coming out? Radioactive ion beams facilities: more data in the pipeline
- Astrophysical interest: decay studies of the $N = 82$ $r$-process
The $^{132}\text{Sn}$ region
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The doubly-magic $^{132}\text{Sn}$ is a robust core
The $^{132}$Sn region

The doubly-magic $^{132}$Sn is a robust core

\[\downarrow\]

Nuclear shell model is the best tool to investigate nuclei with valence particle/hole out/inside $^{132}$Sn
The $^{132}\text{Sn}$ region

The doubly-magic $^{132}\text{Sn}$ is a robust core

Nuclear shell model is the best tool to investigate nuclei with valence particle/hole out/inside $^{132}\text{Sn}$

If shell model works, this region is particularly suited for calculation with realistic $N\!N$ potentials
The $^{132}$Sn region

Realistic shell-model calculations have been widely performed since mid 1990s


The $^{132}$Sn region

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At present, the derivation of realistic shell-model interactions has reached a good quality level, and calculations are able to describe a large set of experimental data
The shell-model effective interaction

Schrödinger equation for $A$ nucleons in the whole Hilbert space:

$$H \Phi_i(1, \ldots, A) = E_i \Phi_i(1, \ldots, A)$$
The shell-model effective interaction

Schrödinger equation for $A$ nucleons in the whole Hilbert space:

$$H\Phi_i(1, \ldots, A) = E_i \Phi_i(1, \ldots, A)$$

Nuclear shell-model scheme: 2 nucleons interacting in the model space $P$

$$PH_{\text{eff}}P\Psi_\alpha = \tilde{E}_\alpha P\Psi_\alpha$$

$$\{\tilde{E}_\alpha\} \in \{E_j\}$$
The shell-model effective interaction

A very useful way to derive $V_{\text{eff}}$ is the time-dependent perturbative approach as developed by Kuo and co-workers in the 1970s (see T. T. S. Kuo and E. Osnes, Lecture Notes in Physics vol. 364 (1990))
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A very useful way to derive $V_{\text{eff}}$ is the time-dependent perturbative approach as developed by Kuo and co-workers in the 1970s (see T. T. S. Kuo and E. Osnes, Lecture Notes in Physics vol. 364 (1990)). In this approach the effective hamiltonian $H_{\text{eff}}$ is expressed as

$$H_{\text{eff}} = \hat{Q} - \hat{Q}' \int \hat{Q} + \hat{Q}' \int \hat{Q} \int \hat{Q} - \hat{Q}' \int \hat{Q} \int \hat{Q} \int \hat{Q} \cdots,$$
The shell-model effective interaction

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- The so-called $\hat{Q}$-box is a collection of irreducible valence-linked diagrams.
- The integral sign represents a generalized folding operation.
The $\hat{Q}$-box

The $\hat{Q}$-box diagrams are divided into a one-body component, called $\hat{S}$-box:
The $\hat{Q}$-box

... and a two-body component:

The shell-model effective interaction

Kuo and Krenciglowa have proved that

\[ \hat{Q} \int \hat{Q} = -\frac{d\hat{Q}(\omega)}{d\omega} \hat{Q}(\omega) \]
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The above relation is the at basis of two iterative techniques to sum up the folded-diagram perturbative series:
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- The Krenciglowa-Kuo partial summation
The shell-model effective interaction

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The above relation is the at basis of two iterative techniques to sum up the folded-diagram perturbative series:

- The Krenciglowa-Kuo partial summation
- The Lee-Suzuki partial summation
Realistic $NN$ potentials

Fundamental ingredients for a realistic shell-model calculation:
Realistic $NN$ potentials

Fundamental ingredients for a realistic shell-model calculation:

- a realistic nucleon-nucleon potential $V_{ij}$
- CD-Bonn
- Argonne V18
- chiral potentials
Realistic $NN$ potentials

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- possibly, a three-body force $V_{ijk}$
  - Urbana IX
  - Illinois 1-5
  - Chiral many-body potentials
Realistic $NN$ potentials

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All the two-body potentials show a “hard core” repulsion in the short-range component: we need to renormalize it
The $G$-matrix

The standard way to renormalize the short-range repulsion is to resort to the theory of the Brueckner reaction matrix $G$. 
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\[
\begin{array}{c}
V_{NN} \\
\end{array}
\begin{array}{c}
a \\
b
\end{array}
\begin{array}{c}
a \\
b
\end{array}
\begin{array}{c}
\rightarrow \\
\text{sm} \\
n
\end{array}
\begin{array}{c}
c \\
d
\end{array}
\begin{array}{c}
c \\
d
\end{array}
\begin{array}{c}
c \\
d
\end{array}
\begin{array}{c}
c \\
d
\end{array}
\begin{array}{c}
c \\
d
\end{array}
\begin{array}{c}
+ \\
+ \\
+ \\
\cdots
\end{array}
\]
The $G$-matrix

The standard way to renormalize the short-range repulsion is to resort to the theory of the Brueckner reaction matrix $G$

It can be written by way of an integral equation:

$$G(ab, cd) = V_{NN}(ab, cd) + \frac{1}{2} \sum_{\alpha\beta} \frac{V_{NN}(ab, \alpha\beta)G(\alpha\beta, cd)}{\epsilon_c + \epsilon_d - \epsilon_\alpha - \epsilon_\beta},$$

or in operatorial form:

$$G(\omega) = V_{NN} + V_{NN} \frac{Q_{2p}}{\omega - H_0} G(\omega)$$
The $V_{\text{low}-k}$

Inspiration to renormalize $V_{NN}$:
The $V_{\text{low} - k}$

Inspiration to renormalize $V_{NN}$:

- Effective field theory (EFT)
- Renormalization group (RG)
The $V_{\text{low}-k}$

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- Effective field theory (EFT)
- Renormalization group (RG)

from EFT: we restrict the configurations of $V_{NN}(k, k')$ to those with $k, k' < k_{\text{cutoff}} = \Lambda$

The $V_{\text{low} - k}$

The deuteron hamiltonian:

In the full space:

$$\langle k | (T + V_{NN}) | k' \rangle \langle k' | \Psi_n \rangle = E_n \langle k | \Psi_n \rangle \quad 0 < k, k' < \infty$$

In a reduced model space $P = \sum_{k<\Lambda} |k\rangle \langle k|$:

$$\langle k | (T + V_{\text{eff}}) | k' \rangle \langle k' | \Phi_m \rangle = \tilde{E}_m \langle k | \Phi_m \rangle \quad 0 < k, k' < \Lambda$$

Fundamental constraint: $\tilde{E}_m \in \{E_n\}$
The $V_{\text{low-k}}$

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How to construct $\langle k | H_{\text{eff}} | k' \rangle$?
The $V_{\text{low} - k}$

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How to construct $\langle k | H_{\text{eff}} | k' \rangle$?

↓

Unitary transformation: Andreozzi-Lee-Suzuki method
The $V_{\text{low}-k}$

**Numerical test:** deuteron binding energy with the CD-Bonn potential

<table>
<thead>
<tr>
<th>$\Lambda$ (in fm$^{-1}$)</th>
<th>$PV_{\text{eff}}P$ (in MeV)</th>
<th>$V_{NN}$ (in MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.6</td>
<td>-2.225</td>
<td>-2.225</td>
</tr>
<tr>
<td>1.8</td>
<td>-2.225</td>
<td></td>
</tr>
<tr>
<td>2.0</td>
<td>-2.225</td>
<td></td>
</tr>
<tr>
<td>2.2</td>
<td>-2.225</td>
<td></td>
</tr>
</tbody>
</table>
The $V_{\text{low}-k}$

Numerical test: phase shifts in the $^1S_0$ channel (in degrees)

<table>
<thead>
<tr>
<th>$E_{\text{lab}}$ (MeV)</th>
<th>CD-Bonn</th>
<th>$V_{\text{low}-k}$</th>
<th>Expt.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>62.1</td>
<td>62.1</td>
<td>62.1</td>
</tr>
<tr>
<td>10</td>
<td>60.0</td>
<td>60.0</td>
<td>60.0</td>
</tr>
<tr>
<td>25</td>
<td>50.9</td>
<td>50.9</td>
<td>50.9</td>
</tr>
<tr>
<td>50</td>
<td>40.5</td>
<td>40.5</td>
<td>40.5</td>
</tr>
<tr>
<td>100</td>
<td>26.4</td>
<td>26.4</td>
<td>26.8</td>
</tr>
<tr>
<td>150</td>
<td>16.3</td>
<td>16.3</td>
<td>16.9</td>
</tr>
<tr>
<td>200</td>
<td>8.3</td>
<td>8.3</td>
<td>8.9</td>
</tr>
<tr>
<td>250</td>
<td>1.6</td>
<td>1.6</td>
<td>2.0</td>
</tr>
<tr>
<td>300</td>
<td>-4.3</td>
<td>-4.3</td>
<td>-4.5</td>
</tr>
</tbody>
</table>
Cut-off momentum $\Lambda$

General criterion

- small enough so to give a reasonably smooth potential suitable to be used directly in a perturbative scheme
- large enough so that $V_{\text{low-k}}$ reproduces the same phase-shifts of the original $V_{NN}$ up to the anelastic threshold ($E_{\text{lab}} \simeq 350$ MeV)

$$E_{\text{lab}} \leq 2\hbar^2 \Lambda^2 / M \rightarrow \Lambda \simeq 2.1 \text{fm}^{-1}$$
Shell-model calculations
Shell-model calculations

Model space outside $^{132}$Sn core

<table>
<thead>
<tr>
<th>proton</th>
<th>Energy (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0g_{7/2}$</td>
<td>0.000 (-9.663)</td>
</tr>
<tr>
<td>$1d_{5/2}$</td>
<td>0.962</td>
</tr>
<tr>
<td>$1d_{3/2}$</td>
<td>2.440</td>
</tr>
<tr>
<td>$2s_{1/2}$</td>
<td>2.800</td>
</tr>
<tr>
<td>$0h_{11/2}$</td>
<td>2.793</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>neutron</th>
<th>Energy (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0h_{9/2}$</td>
<td>1.561</td>
</tr>
<tr>
<td>$1f_{7/2}$</td>
<td>0.000 (-2.461)</td>
</tr>
<tr>
<td>$1f_{5/2}$</td>
<td>2.001</td>
</tr>
<tr>
<td>$2p_{3/2}$</td>
<td>0.864</td>
</tr>
<tr>
<td>$2p_{1/2}$</td>
<td>1.656 (1.5)</td>
</tr>
<tr>
<td>$0i_{13/2}$</td>
<td>2.700 - unbound?</td>
</tr>
</tbody>
</table>
Shell-model calculations

- Experimental single-particle energies
Shell-model calculations

- Experimental single-particle energies
- Input realistic potential: $V_{\text{low-k}}$ derived from the CD-Bonn potential and a cutoff $\Lambda = 2.2 \text{ fm}^{-1}$
Shell-model calculations

- Experimental single-particle energies
- Input realistic potential: $V_{\text{low-k}}$ derived from the CD-Bonn potential and a cutoff $\Lambda = 2.2 \text{ fm}^{-1}$
- Effective interaction derived at 2nd order of the theory
Shell-model calculations

- Experimental single-particle energies
- Input realistic potential: $V_{\text{low-}k}$ derived from the CD-Bonn potential and a cutoff $\Lambda = 2.2 \text{ fm}^{-1}$
- Effective interaction derived at 2nd order of the theory
- Number of intermediate states large enough to obtain stable two-nucleon matrix elements: up to 5 h.o. shells above the Fermi surface
Shell-model calculations

Fundamental test for $V_{\text{eff}}$: spectroscopy of the two-valence system
Shell-model calculations

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Fundamental test for $V_{\text{eff}}$: spectroscopy of the two-valence system

$^{134}\text{Sb}$ proton-neutron multiplets
Shell-model calculations

Test for the quality of the predicted wavefunctions: calculation of the electromagnetic transition rates \( (\epsilon_p^{\text{eff}} = 1.55, \epsilon_n^{\text{eff}} = 0.7) \)
Shell-model calculations

Test for the quality of the predicted wavefunctions: calculation of the electromagnetic transition rates \( (e_p^{\text{eff}} = 1.55, e_n^{\text{eff}} = 0.7) \)

<table>
<thead>
<tr>
<th>(^{134}\text{Te})</th>
<th>Calc.</th>
<th>Expt.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\text{B(E2;}0_1^+ \rightarrow 2_1^+)) (in W.u.)</td>
<td>20</td>
<td>24(3)</td>
</tr>
<tr>
<td>(\text{B(E2;}4_1^+ \rightarrow 2_1^+)) (in W.u.)</td>
<td>4.3</td>
<td>4.3(0.3)</td>
</tr>
<tr>
<td>(\text{B(E2;}6_1^+ \rightarrow 4_1^+)) (in W.u.)</td>
<td>1.9</td>
<td>2.05(0.03)</td>
</tr>
</tbody>
</table>
Shell-model calculations

Test for the quality of the predicted wavefunctions: calculation of the electromagnetic transition rates ($e^\text{eff}_p = 1.55$, $e^\text{eff}_n = 0.7$)

<table>
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<td></td>
</tr>
<tr>
<td></td>
<td>$^{134}\text{Sn}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$B(E2;0^+_1 \to 2^+_1)$ (in $e^2b^2$)</td>
<td>0.033</td>
<td>0.029(4)</td>
<td></td>
</tr>
</tbody>
</table>
Shell-model calculations

Test for the quality of the predicted wavefunctions: calculation of the electromagnetic transition rates \((e_p^{\text{eff}} = 1.55, e_n^{\text{eff}} = 0.7)\)

\[
\begin{array}{|c|c|c|}
\hline
\text{Nuc.} & \text{Calc.} & \text{Expt.} \\
\hline
^{134}\text{Te} & & \\
\text{B}(\text{E2;}^{0^+_1 \rightarrow 2^+_1}) \ (\text{in W.u.}) & 20 & 24(3) \\
\text{B}(\text{E2;}^{4^+_1 \rightarrow 2^+_1}) \ (\text{in W.u.}) & 4.3 & 4.3(0.3) \\
\text{B}(\text{E2;}^{6^+_1 \rightarrow 4^+_1}) \ (\text{in W.u.}) & 1.9 & 2.05(0.03) \\
\hline
^{134}\text{Sn} & & \\
\text{B}(\text{E2;}^{0^+_1 \rightarrow 2^+_1}) \ (\text{in } e^2 b^2) & 0.033 & 0.029(4) \\
\hline
^{134}\text{Sb} & & \\
\text{B}(\text{E2;}^{2^-_1 \rightarrow 0^-_1}) \ (\text{in } e^2 \text{fm}^4) & 123 & 429(238) \\
\text{B}(\text{E2;}^{3^-_1 \rightarrow 1^-_1}) \ (\text{in } e^2 \text{fm}^4) & 115 & 118(26) \\
\text{B}(\text{M1;}^{3^-_1 \rightarrow 2^-_1}) \ (\text{in nm}^2) & 1.4 & 2.0(4) \\
\hline
\end{array}
\]
Shell-model calculations

Calculations with effective interactions derived starting from a CD-Bonn $G$-matrix
Shell-model calculations

Calculations with effective interactions derived starting from a CD-Bonn $G$-matrix


<table>
<thead>
<tr>
<th>$J^+$</th>
<th>Experiment</th>
<th>CD-Bonn</th>
</tr>
</thead>
<tbody>
<tr>
<td>0$^+$</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>2$^+$</td>
<td>1.28</td>
<td>1.21</td>
</tr>
<tr>
<td>4$^+$</td>
<td>1.57</td>
<td>1.48</td>
</tr>
<tr>
<td>6$^+$</td>
<td>1.69</td>
<td>1.61</td>
</tr>
<tr>
<td>6$^2_1$</td>
<td>2.40</td>
<td>2.17</td>
</tr>
<tr>
<td>2$^2_2$</td>
<td>2.46</td>
<td>2.45</td>
</tr>
<tr>
<td>4$^2_2$</td>
<td>2.55</td>
<td>2.45</td>
</tr>
<tr>
<td>1$^+$</td>
<td>2.63</td>
<td>2.41</td>
</tr>
<tr>
<td>3$^+$</td>
<td>2.68</td>
<td>2.54</td>
</tr>
<tr>
<td>5$^+$</td>
<td>2.73</td>
<td>2.54</td>
</tr>
<tr>
<td>0$^2_2$</td>
<td>2.66</td>
<td></td>
</tr>
<tr>
<td>3$^2_3$</td>
<td>2.93</td>
<td>3.06</td>
</tr>
</tbody>
</table>
Shell-model calculations

Calculations with effective interactions derived starting from a CD-Bonn $G$-matrix

M. P. Kartamyshev, T. Engeland, M. Hjorth-Jensen, and E. Osnes,

Shell-model calculations

Calculations with effective interactions derived starting from a CD-Bonn $G$-matrix


FIG. 10. Comparison of experimental levels of $^{134}$Sb with shell-model calculations using CD-Bonn, XH208, and XH508 interactions. In (a) are levels observed in the $\beta^-$ decay of $^{134}$Sr, and in (b) are levels populated via $\beta^-$ decay of $^{134}$Sr, along with high-spin levels identified in a fission fragment study [11].
**Shell-model calculations**

**$G$-matrix**

---

**Table:**

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Energy (keV)</th>
<th>Spin/Parity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1207</td>
<td>7/2+</td>
<td></td>
</tr>
<tr>
<td>1117</td>
<td>15/2+</td>
<td></td>
</tr>
<tr>
<td>1113</td>
<td>5/2+</td>
<td></td>
</tr>
<tr>
<td>1027</td>
<td>9/2+</td>
<td></td>
</tr>
<tr>
<td>1014</td>
<td>7/2+</td>
<td></td>
</tr>
<tr>
<td>798</td>
<td>9/2+</td>
<td></td>
</tr>
<tr>
<td>707</td>
<td>11/2+</td>
<td></td>
</tr>
<tr>
<td>640</td>
<td>3/2+</td>
<td></td>
</tr>
<tr>
<td>527</td>
<td>1/2+</td>
<td></td>
</tr>
<tr>
<td>408</td>
<td>3/2+</td>
<td></td>
</tr>
<tr>
<td>318</td>
<td>5/2+</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>7/2+</td>
<td></td>
</tr>
</tbody>
</table>

For $^{135}\text{Sb}$:
- Experimental levels
- $\text{CD Bonn}$ calculations

Fig. 9: Comparison of the experimental levels up to 1200 keV for $^{129}\text{Sb}$ with shell-model calculations using the $\text{CD Bonn}$ interaction with a $\omega_{xy}$ single particle energy lowered by 300 keV. Shown to the far right is the known level structure of $^{134}\text{Sn}$. 
Shell-model calculations

$V_{\text{low-}k}$

Diagram showing energy levels labeled as $\frac{1}{2}^+$, $\frac{3}{2}^+$, $\frac{5}{2}^+$, etc., corresponding to experimental (Expt.) and calculated (Calc.) values for $^{135}\text{Sb}$. The energy scale (E(MeV)) ranges from 0 to 2 MeV.
Shell-model calculations

Main difference the two calculations: with the $V_{\text{low}-k}$ a larger number of intermediate states can be included
Shell-model calculations

Main difference the two calculations: with the $V_{\text{low}-k}$ a larger number of intermediate states can be included

Employing the same reduced $\hat{Q}$-space of intermediate states in the calculation of the $\hat{Q}$-box diagrams, the quality of the results is similar.
Shell-model calculations

$^{134}$Sb proton-neutron multiplets
Shell-model calculations

$^{134}$Sb proton-neutron multiplets
Shell-model calculations

![Diagram showing levels and energies for $^{136}$Th, comparing experimental (Expt.) and calculated (Calc.) results.](image)
Shell-model calculations

\[ B(E2; 0_1^+ \rightarrow 2_1^+) \text{ (in } e^2b^2) \]

<table>
<thead>
<tr>
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<tbody>
<tr>
<td>[ B(E2; 0_1^+ \rightarrow 2_1^+) ] (in ( e^2b^2 ))</td>
<td>0.18</td>
<td>0.103(15)</td>
</tr>
</tbody>
</table>
Shell-model calculations

\[ B(E2; 0^+_1 \rightarrow 2^+_1) \text{ (in } e^2b^2) \]

<table>
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<tr>
<th></th>
<th>Calc.</th>
<th>Expt.</th>
</tr>
</thead>
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<td>( B(E2; 0^+_1 \rightarrow 2^+_1) )</td>
<td>0.18</td>
<td>0.15(1) new!</td>
</tr>
</tbody>
</table>
Shell-model calculations

$^{136}$Te interesting feature: low-lying mixed symmetry states

$$
|^{136}\text{Te}; 2_{1}^{+}\rangle = +0.82|D_{\nu} \times S_{\pi}\rangle + 0.45|S_{\nu} \times D_{\pi}\rangle
$$

$$
|^{136}\text{Te}; 2_{2}^{+}\rangle = +0.38|D_{\nu} \times S_{\pi}\rangle - 0.76|S_{\nu} \times D_{\pi}\rangle
$$

Shell-model calculations

$^{136}$Te interesting feature: low-lying mixed symmetry states

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Using $V_{\text{low-k}}$:

\[ |^{136}\text{Te}; 2^+_1 \rangle = +0.73 |^{134}\text{Te}; \text{g.s.} \rangle |^{134}\text{Sn}; 2^+_1 \rangle + 0.36 |^{134}\text{Sn}; \text{g.s.} \rangle |^{134}\text{Te}; 2^+_1 \rangle \]
\[ |^{136}\text{Te}; 2^+_3 \rangle = -0.31 |^{134}\text{Te}; \text{g.s.} \rangle |^{134}\text{Sn}; 2^+_1 \rangle + 0.78 |^{134}\text{Sn}; \text{g.s.} \rangle |^{134}\text{Te}; 2^+_1 \rangle \]
Shell-model calculations

Calculations for nuclei with many valence-nucleons: the $N = 82$ isotonic chain
Shell-model calculations

Calculations for nuclei with many valence-nucleons: the $N = 82$ isotonic chain

![Graph 1: J^\pi=0^- 2nd excited states](image1.png)

![Graph 2: J^\pi=2^- 1st excited states](image2.png)
Shell-model calculations

The monopole component of the effective interaction: ground state energies as a function of the valence protons
Shell-model calculations

The monopole component of the effective interaction: ground state energies as a function of the valence protons

![Graph showing ground state energy as a function of valence protons](image)
Shell-model calculations

The monopole component of the effective interaction: ground state energies as a function of the valence protons

![Graph showing ground-state energy as a function of valence protons for different models: Expt., CD-Bonn Λ=2.2 fm⁻¹ 2nd order, CD-Bonn Λ=2.2 fm⁻¹ 3rd order.](image)
Single-particle properties

Hot topic: the role of tensor-force contribution to determine the single-particle spacings, and consequently the evolution of shell structure of nuclei

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Realistic potentials include one-pion exchange and more: are effective realistic interactions able to reproduce single-particle spacings?
Preliminary results

Comparison between theoretical and experimental $^{133}$Sb single-particle relative spectra
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Comparison between theoretical and experimental $^{133}\text{Sb}$ single-particle relative spectra
Effective interactions derived from modern $NN$ potentials are able to describe with **quantitative accuracy** the spectroscopy of exotic nuclei near closed shells. This gives confidence in their predictive power in these regions.

At present, **no real evidence** of shell model modifications near $^{132}\text{Sn}$

It is of **key importance** to gain more experimental information for nuclei around $^{100}\text{Sn}$ and $^{132}\text{Sn}$. This is an exciting physics to be done with RIB’s.
Collaborators

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