Equation of state of nuclear matter and nuclei in laboratories and in neutron-star crusts

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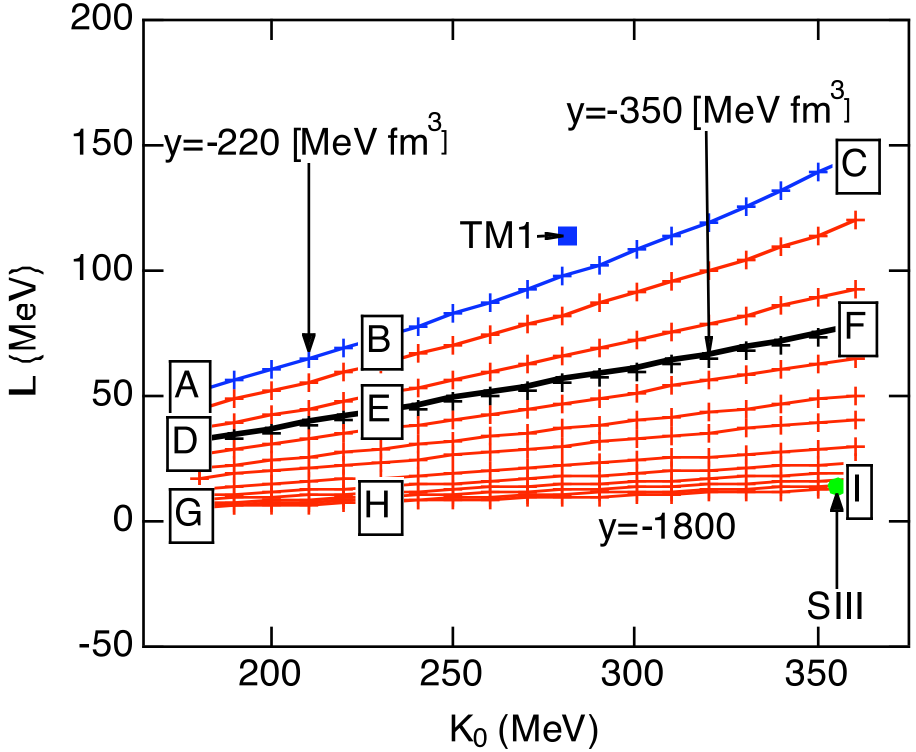
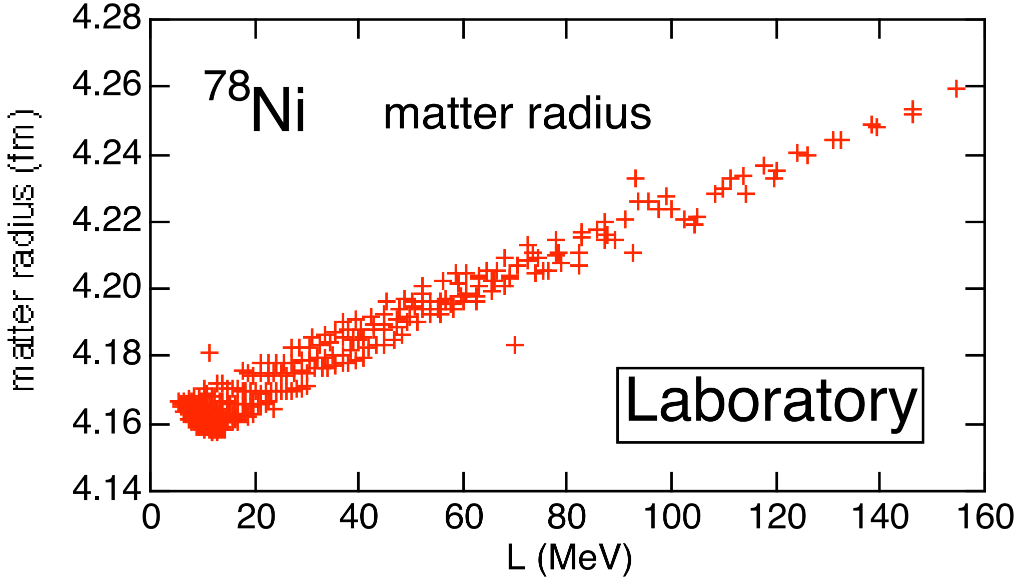
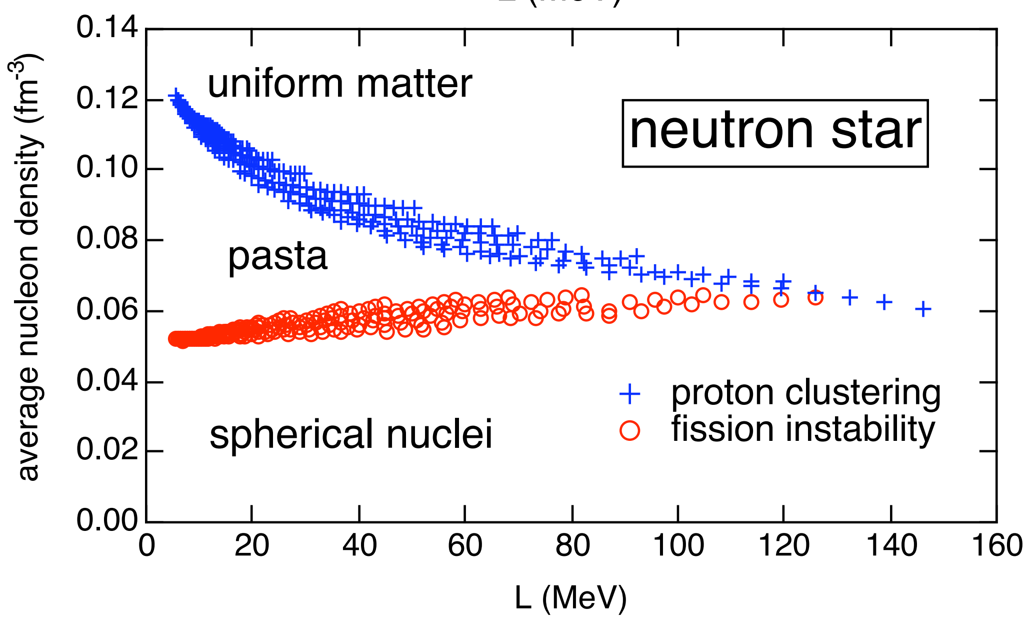
LARGEST UNCERTAINTY

L : density-derivative coefficient of symmetry energy







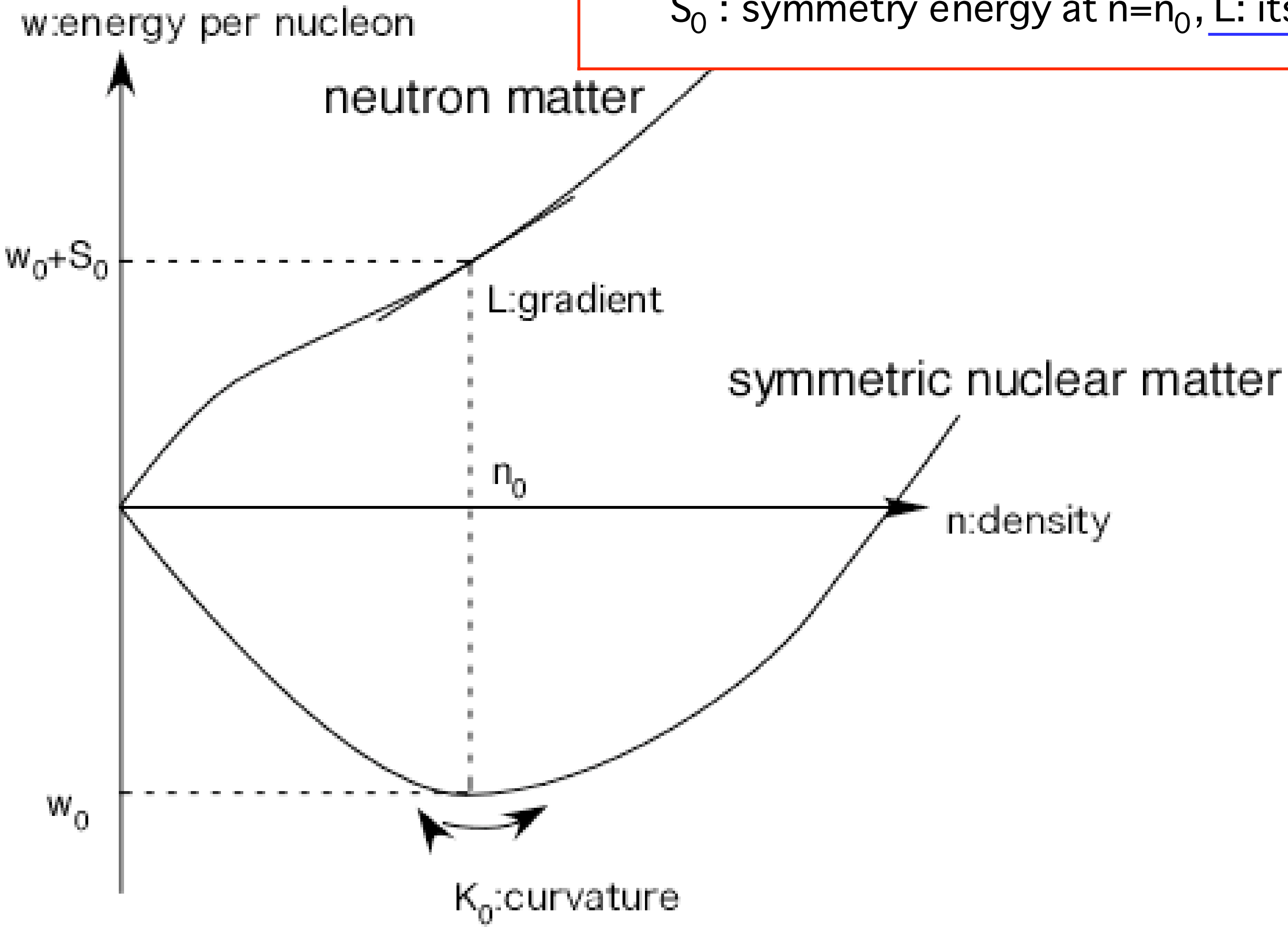
Which EOS parameter dominates macroscopic properties of neutron-rich nuclei in laboratory and in neutron-star crusts?

Energy per nucleon of nearly symmetric nuclear matter

$$w(n, x) \approx w_0 + \frac{K_0}{18 n_0^2}(n - n_0)^2 + (1 - 2x)^2\left[S_0 + \frac{L}{3n_0}(n - n_0)\right]$$

n_0 : nuclear density, w_0 :saturation energy, K_0 : incompressibility

S_0 : symmetry energy at $n=n_0$, L: its density derivative coefficient



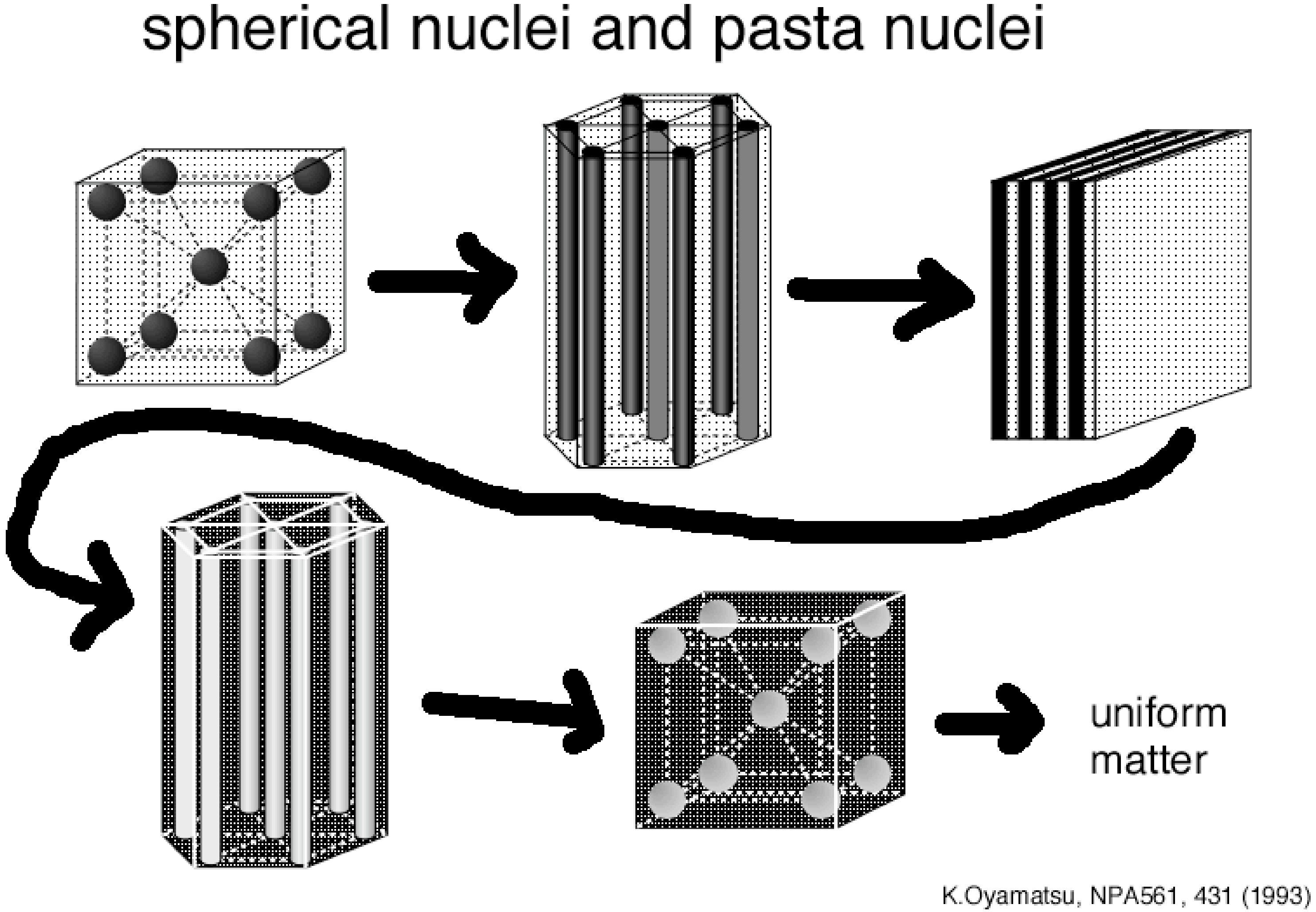
Macroscopic properties of nuclei in labs. and in neutron stars

=> We adopt a macroscopic nuclear model.

Step 1.
From masses and radii of stable nuclei, we construct about 200 EOS's systematically and identify the allowed region of these EOS parameter values.

Step 2.
We calculate masses and radii of unstable nuclei in laboratories and examine which is key EOS parameter.

Step 3.
We calculate nuclei in neutron-star crusts and examine which is key EOS parameter.
Specifically, we show lower and upper boundary densities of pasta-nucleus phase.



Dark domains means nuclei (proton clusters).
At low densities in neutron-star crusts, we have nuclei which are more or less spherical.
In the core we have uniform matter. Pasta nuclei could exists in between.

Existence of pasta nuclei depends on the EOS.

Step 1

Generate all empirically allowed EOS's systematically

K. Oyamatsu and K. Iida, Prog. Theor. Phys. 109, 631 (2003).

Adopted macroscopic model

Energy per cell

$$W = \int_{cell} d\mathbf{r} \left[\varepsilon_0(n_n, n_p) + m_n n_n + m_p n_p \right] + \int_{cell} d\mathbf{r}\, F_0 \left| \nabla n_n \right|^2 + (electron\ kinetic\ energy) + (Coulomb)$$

n_n (n_p) : local neutron (proton) density, $n = n_n + n_p$: total density

$\varepsilon_0(n_n, n_p)$: EOS of uniform nuclear matter (energy density)

F_0 : surafce energy parameter

Parametrization of the EOS (energy density)

$$\varepsilon_0(n_n, n_p) = \underbrace{\frac{3}{5}\left(3\pi^2\right)^{2/3}\left(\frac{\hbar^2}{2m_n} n_n^{5/3} + \frac{\hbar^2}{2m_p} n_p^{5/3}\right)}_{\text{Fermi kinetic energy density}} + \underbrace{\left[1 - (1 - 2Y_p)^2\right] v_s(n) + (1 - 2Y_p)^2 v_n(n)}_{\text{potential energy density}}$$

potential energy densities of symmetric and neutron matter

$$v_s(n) = a_1 n^2 + \frac{a_2 n^3}{1 + a_3 n} \qquad v_n(n) = b_1 n^2 + \frac{b_2 n^3}{1 + b_3 n}$$

★$a_1 \sim b_2$ and F_0 : masses and radii of stable nuclei ($b_3 = 1.59$ fm^3, a fit to FP EOS)

★very flexible function form: a_3 can vary K_0 widely. (better than Skyrme)

The function can be fitted to SIII and TM1 EOS very well.

Simplified Thomas-Fermi calculation

energy minimization with respect to parameters of $n_n(r)$ and $n_p(r)$ (and lattice constant)

neutron (proton) density distribution n_n (n_p)

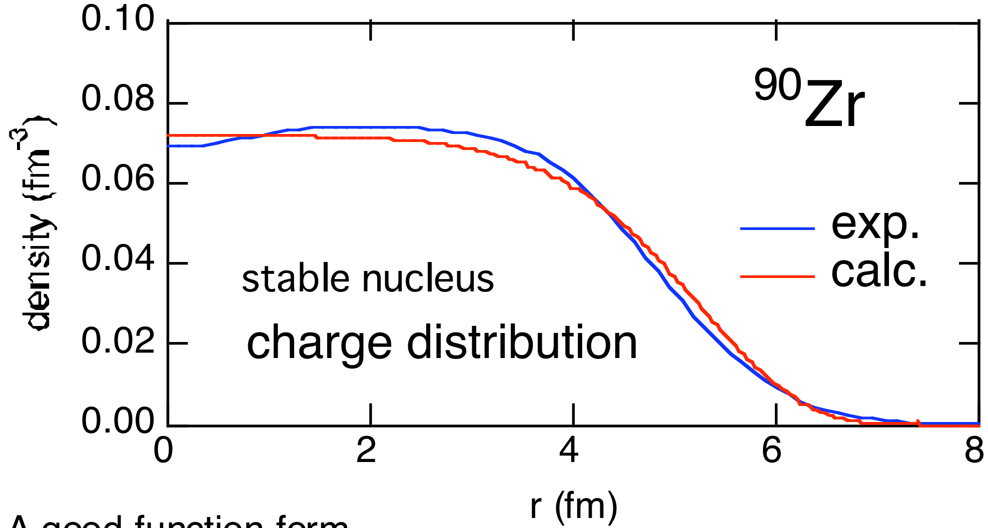
$$n_i(r) = \begin{cases} \left(n_i^{in} - n_i^{out}\right)\left[1 - \left(\frac{r}{R_i}\right)^{t_i}\right]^3 + n_i^{out} & r < R_i \\ n_i^{out} & r < R_i \end{cases}$$

R_n (R_p) : neutron (proton) radius parameter

t_n (t_p) : neutron (proton) surface thickness parameter

n_i^{in} : central desnity

n_n^{out} : neutron gas density (n_p^{out}=0)



A good function form

The n and p distributions are independent.

=> neutron skin

The empirical information is limited: radius and thickness.

The gradient term in Euler Eq. is continuous.

The density is zero beyond the classical turning point.

The values of parameters $a_1 \sim b_3$(EOS) and F_0 are determined

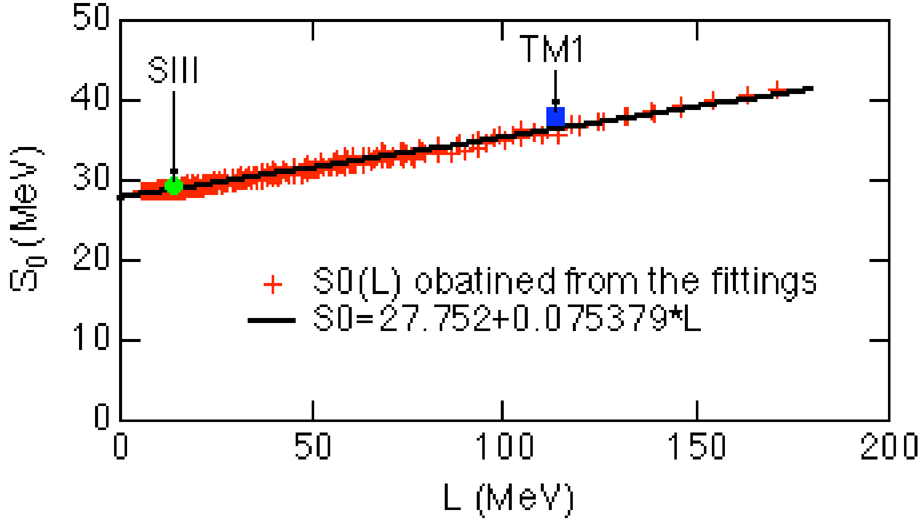
to fit masses and radii of stable nuclei.

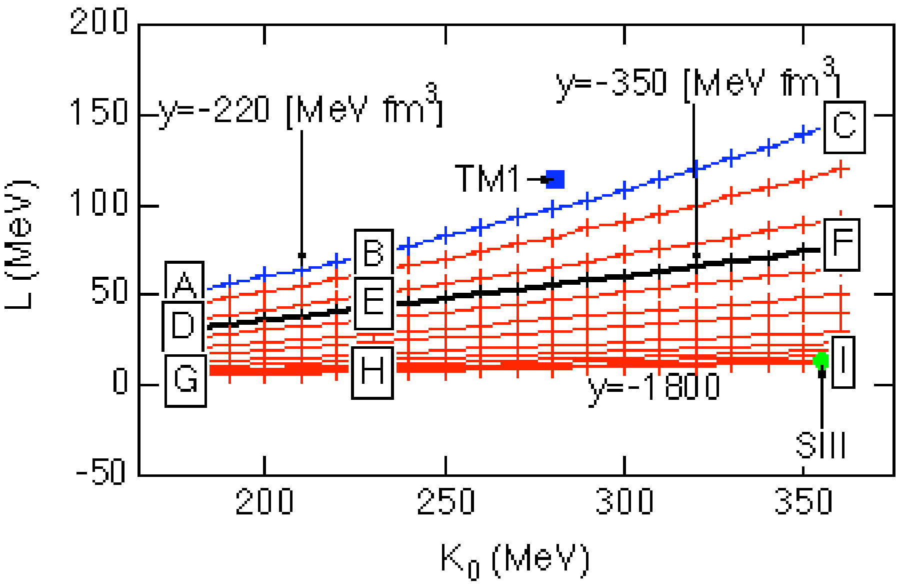
=> about 200 sets of empirical EOS+F_0

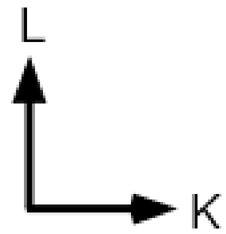
EOS parameter values obtained from stable nuclei

S_0:symmetry energy

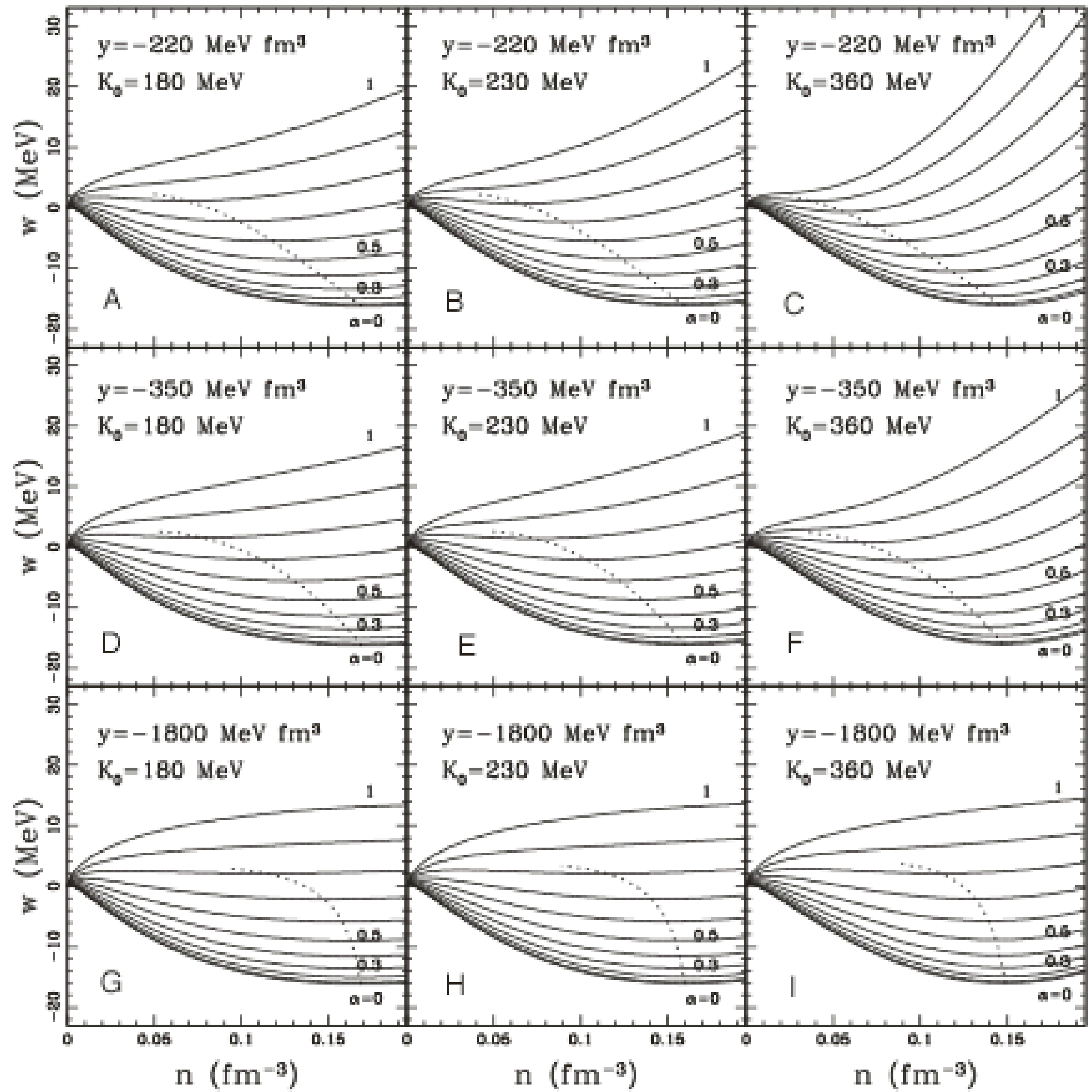
L : density symmetry coefficient







9 representative EOS A-I

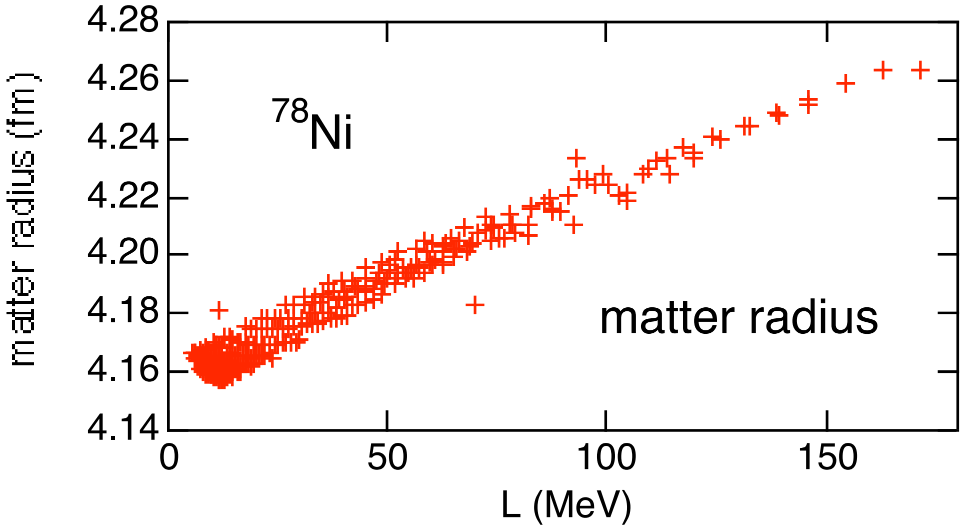


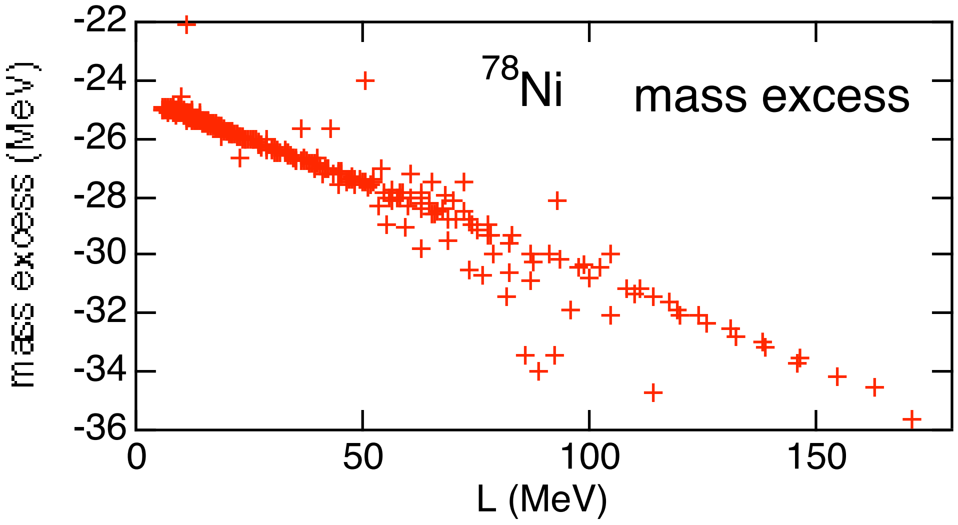
Step 2

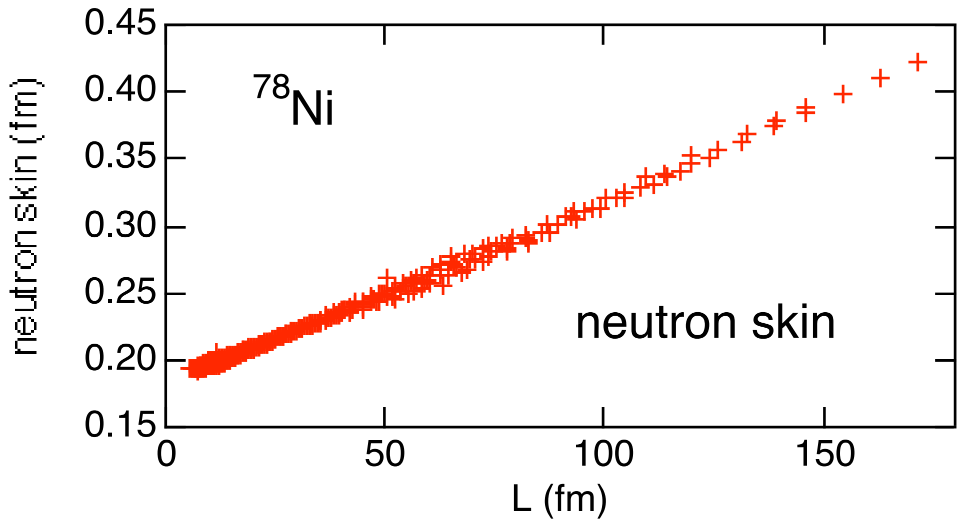
Calculate neutron-rich nuclei in labs with the 200 EOS's

K. Oyamatsu and K. Iida, Prog. Theor. Phys. 109, 631 (2003).

The mass, radius and neutron skin are dependent on L but not on K_0.







Step 3

Calculate nuclei in neutron star crusts with 200 EOS's

Density region of pasta nuclei

K. Oyamatsu and K. Iida, Phys. Rev. **C75** (2007) 015801.

Estimate of density region of pasta nuclei

C.J. Pethick and D.G. Ravenhall, Annu. Rev. Nucl. Part. Sci.**45**, 429 (1995).

lower boundary

stability against fission of spherical nuclei

In the liquid drop model, (Coulomb self energy)=2*(surface energy)

==> (volume fraction of nucleus) = 1/8

upper boundary (core-crust boundary)

instability against forming proton clusters

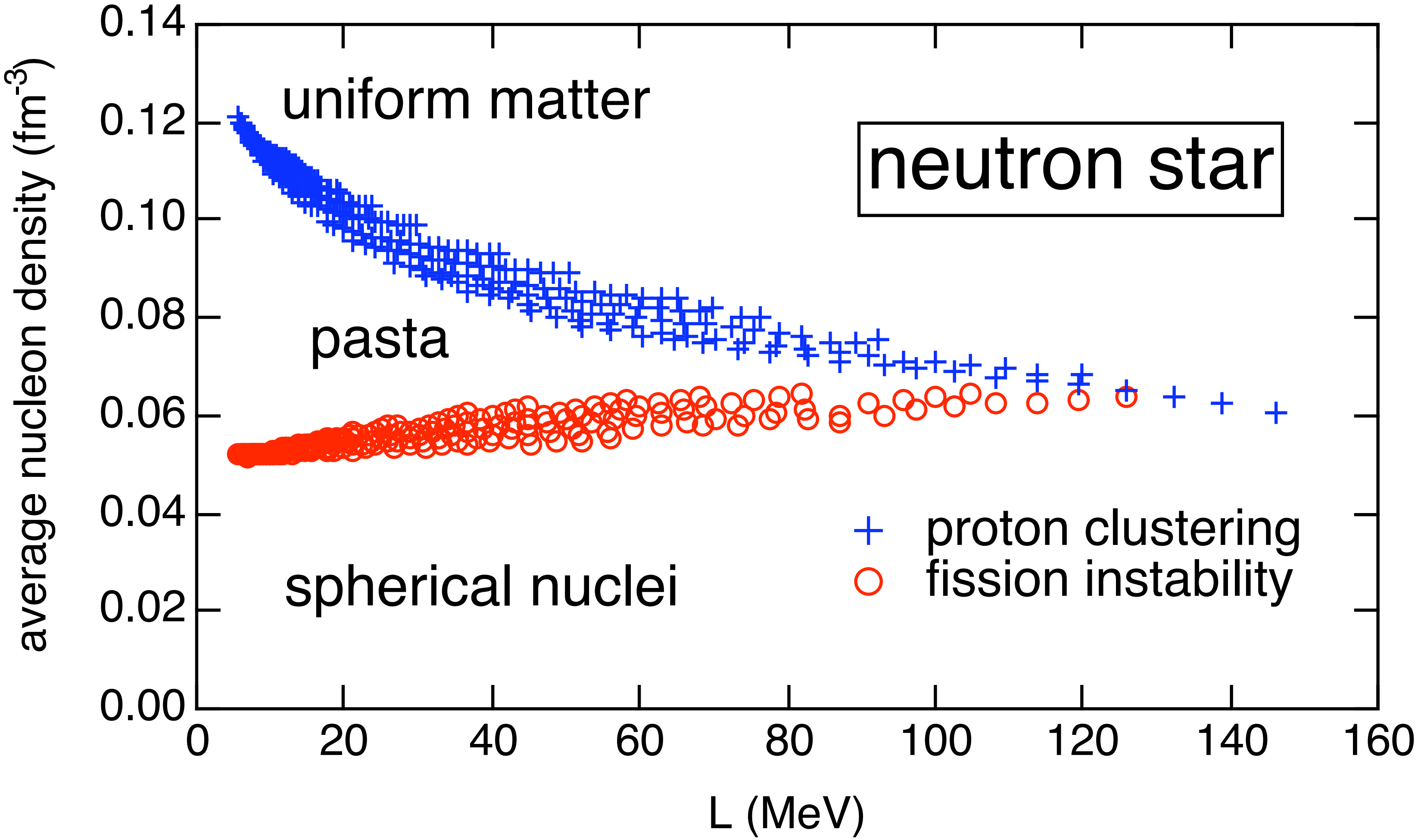
$$v(Q) = v_0 + 2(4\pi e^2\beta)^{1/2} - \beta k_{TF}^2 > 0 \qquad Q^2 = \left(\frac{4\pi e^2}{\beta}\right)^{1/2} - k_{TF}^2$$

$$v(Q) \approx v_0 = \frac{\partial\mu_p}{\partial n_p} - \frac{(\partial\mu_p/\partial n_n)^2}{\partial\mu_n/\partial n_n} = \left(\frac{\partial\mu_p}{\partial n_p}\right)_{\mu_n,\mu_e}$$

$$\beta = D_{pp} + 2D_{np}\zeta + D_{nn}\zeta^2, \qquad \zeta = -\frac{\partial\mu_p/\partial n_n}{\partial\mu_n/\partial n_n},$$

$$k_{TF}^2 = \frac{4\pi e^2}{\partial\mu_e/\partial n_e} = \frac{4\alpha}{\pi}(3\pi^2 n_e)^{1/3}.$$

The upper bound (core-crust boundary density) is clearly dependent on L while the lower is almost constant.



Summary

- The values of L and K_0 cannot be determined from masses and radii of stable nuclei.
- Radii and masses of unstable nuclei have appreciable sensitivity to L.
- The core-crust boundary density of neutron star is dependent on L.
- The existence of the pasta phase is dominated by L. The pasta phase exists if $L < 100$ MeV.
- The present uncertainty in L is too large.
- Systematic experimental study of nuclear mass and size of unstable nuclei in laboratories will help determine the L value and the existence of pasta nuclei in neutron stars.