Hyperon-Quark Mixed Phase in Compact Stars

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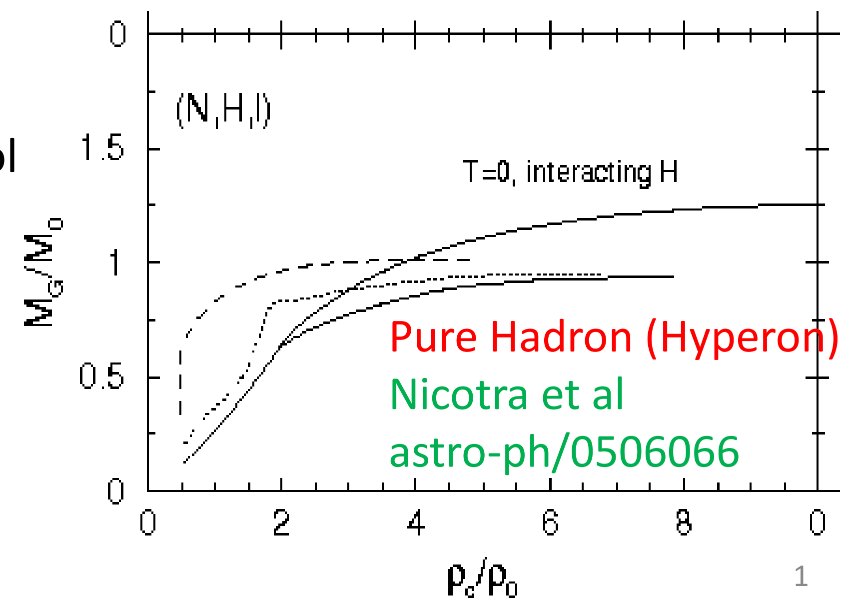
- Non uniform structure and EOS of high density matter.
- Structure and mass of compact stars.

At 2-3 ρ_0, hyperons are expected to appear.

→ Softening of EOS

→ Maximum mass of neutron star becomes less than 1.4 solar mass.

→ Contradicts the obs >1.5 M_{sol}

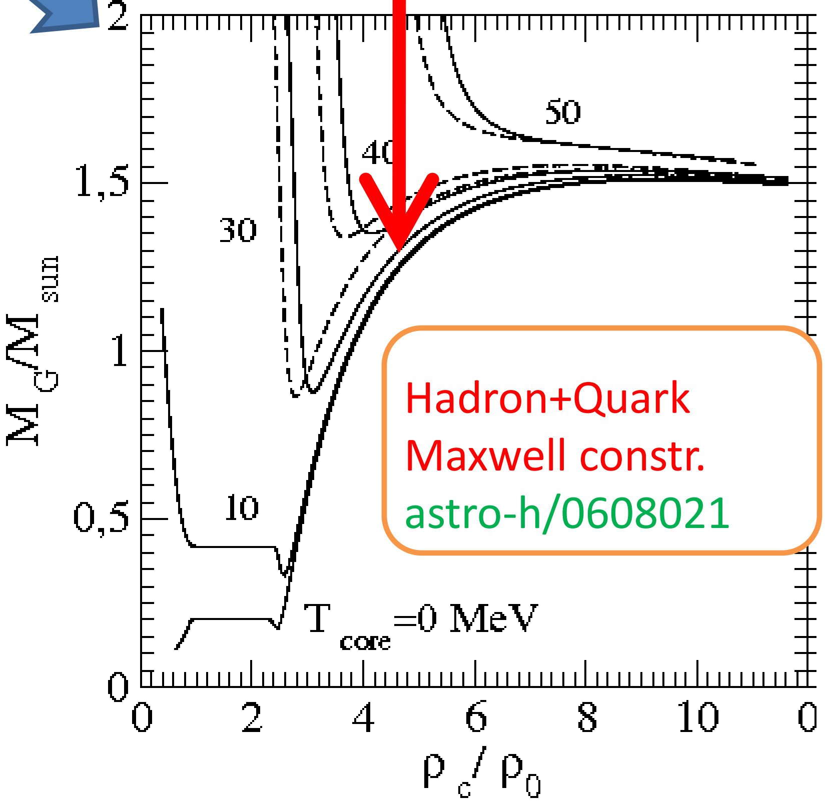


Phase transition to quark matter may solve this problem. [Catania group, PLB562,153 etc].

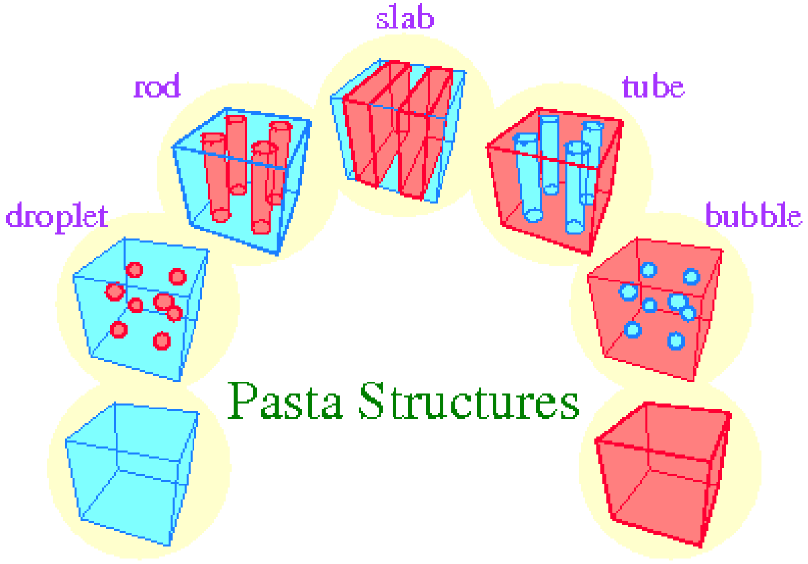
But the existence of mixed phase may soften the EOS again!

Bulk Gibbs calculation yields wide range of mixed phase and large softening [Glendenning, PRD46,1274].

$M_{max}(T=0) \approx 1.5 M_{sun}$



In the mixed phase with charged particles, non-uniform ``Pasta'' structures are expected.



Depending on the density, geometrical structure of mixed phase changes from droplet, rod, slab, tube and to bubble configuration.

Quite different from a bulk picture of mixed phase. We have to take into account the effect of the structure when we calculate EOS.

Hadron phase

Brueckner Hartree Fock model

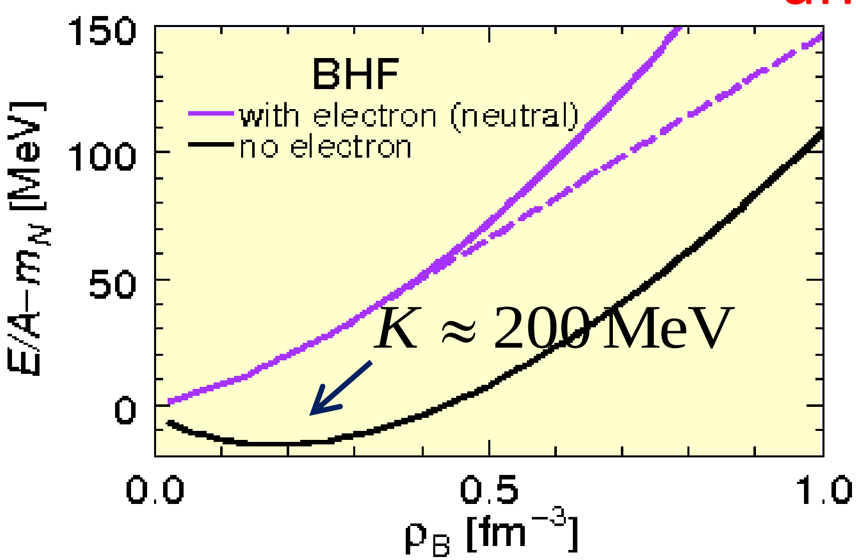
$$\varepsilon = \frac{3}{5}\frac{k_F^2}{2m}\rho + \frac{1}{2}\sum_{k,k'\le k_F}\langle kk'|\mathcal{A}G[\rho;e(k)+e(k')]|kk'\rangle \quad (\mathcal{A}:\text{antisymmetrizer})$$

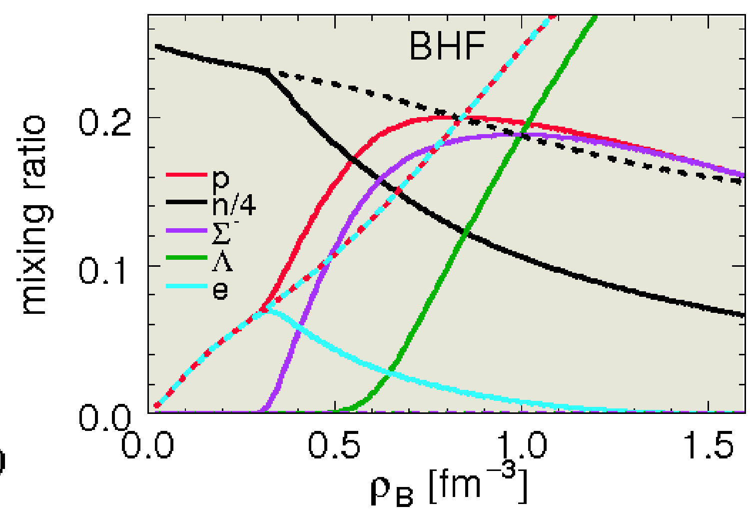
$$G[\rho;w] \equiv v + \sum_{k_a,k_b} v \frac{|k_a k_b\rangle Q\langle k_a k_b|}{w - e(k_a) - e(k_b)} G[\rho;w]$$

$$e(k) = e(k;\rho) \equiv \frac{k^2}{2m} + \mathrm{Re}\sum_{k'\le k_F}\langle kk'|\mathcal{A}G[\rho;e(k)+e(k')]|kk'\rangle$$

v : AV18 + UIX + NSC89

YN scatt data, Λ-nuclear levels, and nuclear saturation are fitted.





Quark phase

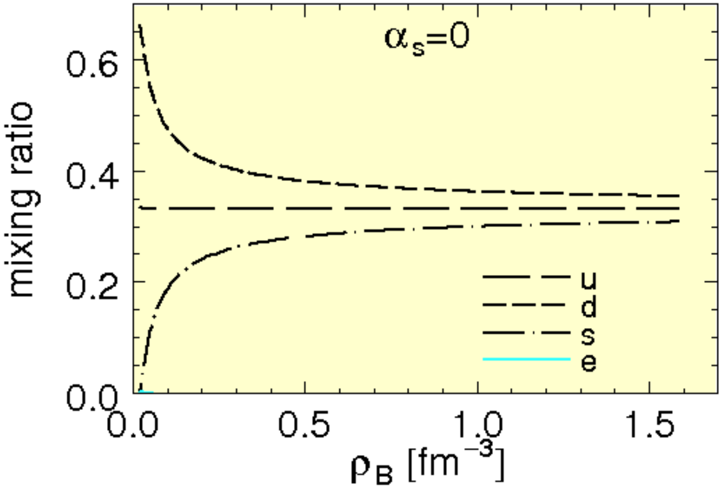
MIT bag model $$\varepsilon = \sum_f \left[\varepsilon_f^{\mathrm{kin}} + \varepsilon_f^{\mathrm{Fock}}\right] + B \qquad (B : \text{bag constant})$$

$$\varepsilon_f^{\mathrm{kin}} = \frac{3}{8\pi^2} m_f^4 \left[x_f \eta_f \left(2x_f^2 + 1\right) - \log\left(x_f + \eta_f\right)\right]$$

$$\varepsilon_f^{\mathrm{Fock}} = \frac{\alpha_s}{8\pi^3} m_f^4 \left[x_f^4 - \frac{3}{2}\left[x_f \eta_f - \log\left(x_f + \eta_f\right)\right]^2\right]$$

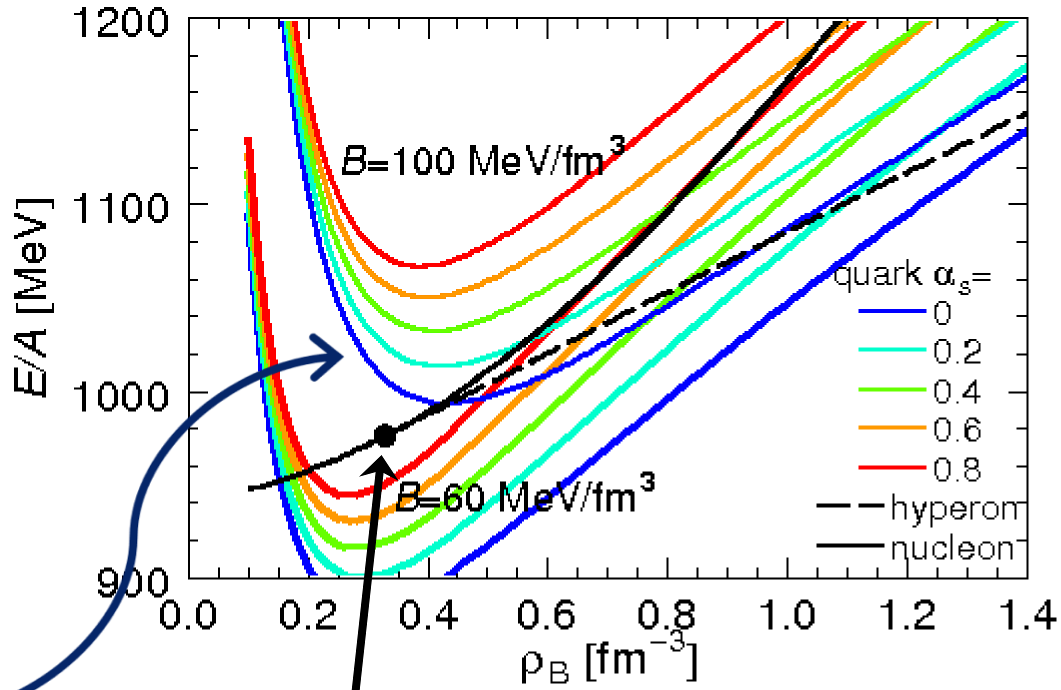
$$x_f \equiv p_F^{(f)} / m_f , \quad \eta_f \equiv \sqrt{1 + x_f^2}$$

Electron fraction is very small in quark matter.



Hadron EOS vs quark EOS

Depending on B and α_S, hadron and quark EOSs cross at various densities.



We choose α_S =0 and B=100 MeV/fm^2.
Quark threshold is above the hyperon threshold in uniform matter.

Hyperon threshold
0.34 fm^{-3}

Coupled equations

to get density profile, energy, pressure, etc of the system

$$\mu_u + \mu_e = \mu_d = \mu_s, \quad \mu_n = \mu_u + 2\mu_d, \quad \mu_p + \mu_e = \mu_n = \mu_\Lambda = \mu_\Sigma - \mu_e$$

$$\mu_i = \frac{\partial \varepsilon(\mathbf{r})}{\partial \rho_i(\mathbf{r})} \quad (i = u, d, s, p, n, \Lambda, \Sigma^-, e)$$

$$\varepsilon(\mathbf{r}) \equiv \varepsilon_B(\mathbf{r}) + \varepsilon_e(\mathbf{r}) + \left(\nabla V_{\mathrm{C}}(\mathbf{r})\right)^2 / 8\pi e^2$$

$$\varepsilon_B(\mathbf{r}) = \begin{cases} \varepsilon_H(\mathbf{r}) & \text{(hadron phase)} \\ \varepsilon_Q(\mathbf{r}) & \text{(quark phase)} \end{cases}$$

$$\varepsilon_e(\mathbf{r}) = \left(3\pi^2 \rho_e(\mathbf{r})\right)^{4/3} / 4\pi^2$$

$$E/A = \frac{1}{\rho_B V}\left[\int_V d^3 r\, \varepsilon(\mathbf{r}) + \sigma S\right] \quad \begin{pmatrix} \rho_B = \text{average baryon density} \\ S = \text{Q-H boundary area} \\ V = \text{cell volume} \end{pmatrix}$$

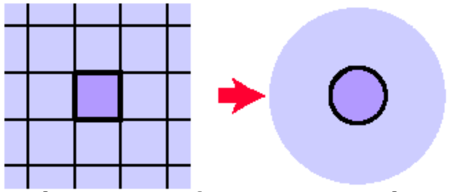
$$\int_V d^3 r \left[\rho_p(\mathbf{r}) - \rho_\Sigma(\mathbf{r}) + \frac{2}{3}\rho_u(\mathbf{r}) - \frac{1}{3}\rho_d(\mathbf{r}) - \frac{1}{3}\rho_s(\mathbf{r}) - \rho_e(\mathbf{r})\right] = 0 \quad \text{(total charge)}$$

$$\frac{1}{V}\int_V d^3 r \left[\rho_p(\mathbf{r}) + \rho_n(\mathbf{r}) + \rho_\Lambda(\mathbf{r}) + \rho_\Sigma(\mathbf{r}) + \frac{1}{3}\rho_u(\mathbf{r}) + \frac{1}{3}\rho_d(\mathbf{r}) + \frac{1}{3}\rho_s(\mathbf{r})\right] = \rho_B \quad \text{(given)}$$

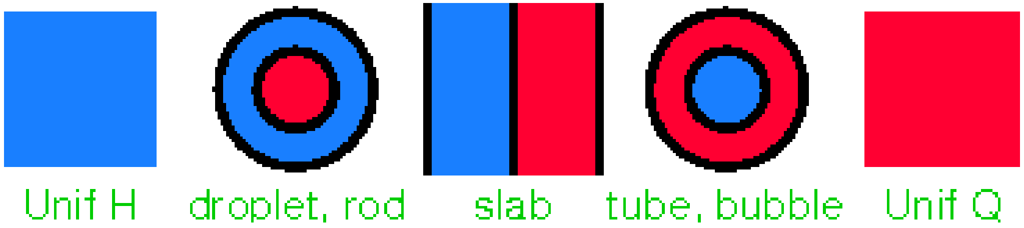
Numerical calculation

• Assume regular structures: Divide space into equivalent and charge-neutral cells with a geometrical symmetry (3D: sphere, 2D : cylinder, 1D: plate).

→ Wigner Seitz cell approximation



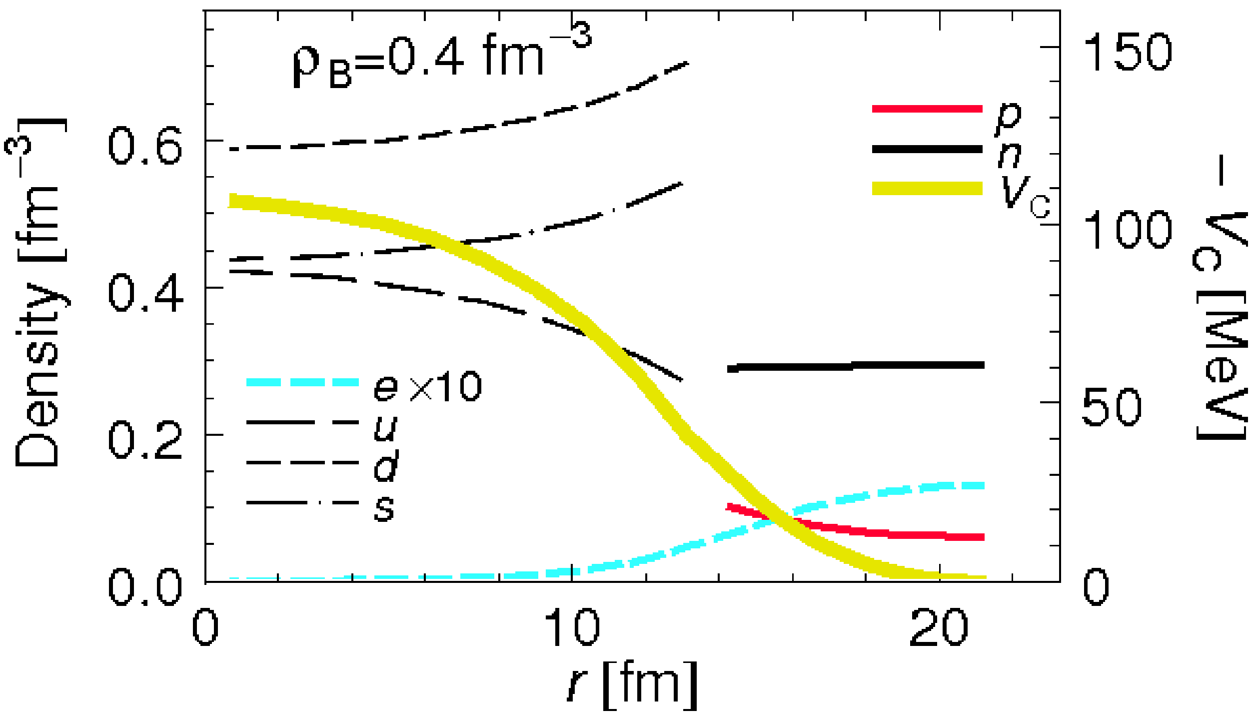
• Put a Q-phase, a H-phase and a phase boundary in the cell.



• Divide the cell into grid points and solve the field equations with a given Baryon density. Cell radius and boundary position are optimized.

• Compare 7 cases (Uniform H, 3D Q-droplet, 2D Q-rod, 1D slab, 2D tube, 3D bubble, Uniform Q) and choose the energy minimum solution.

Density profile in a cell



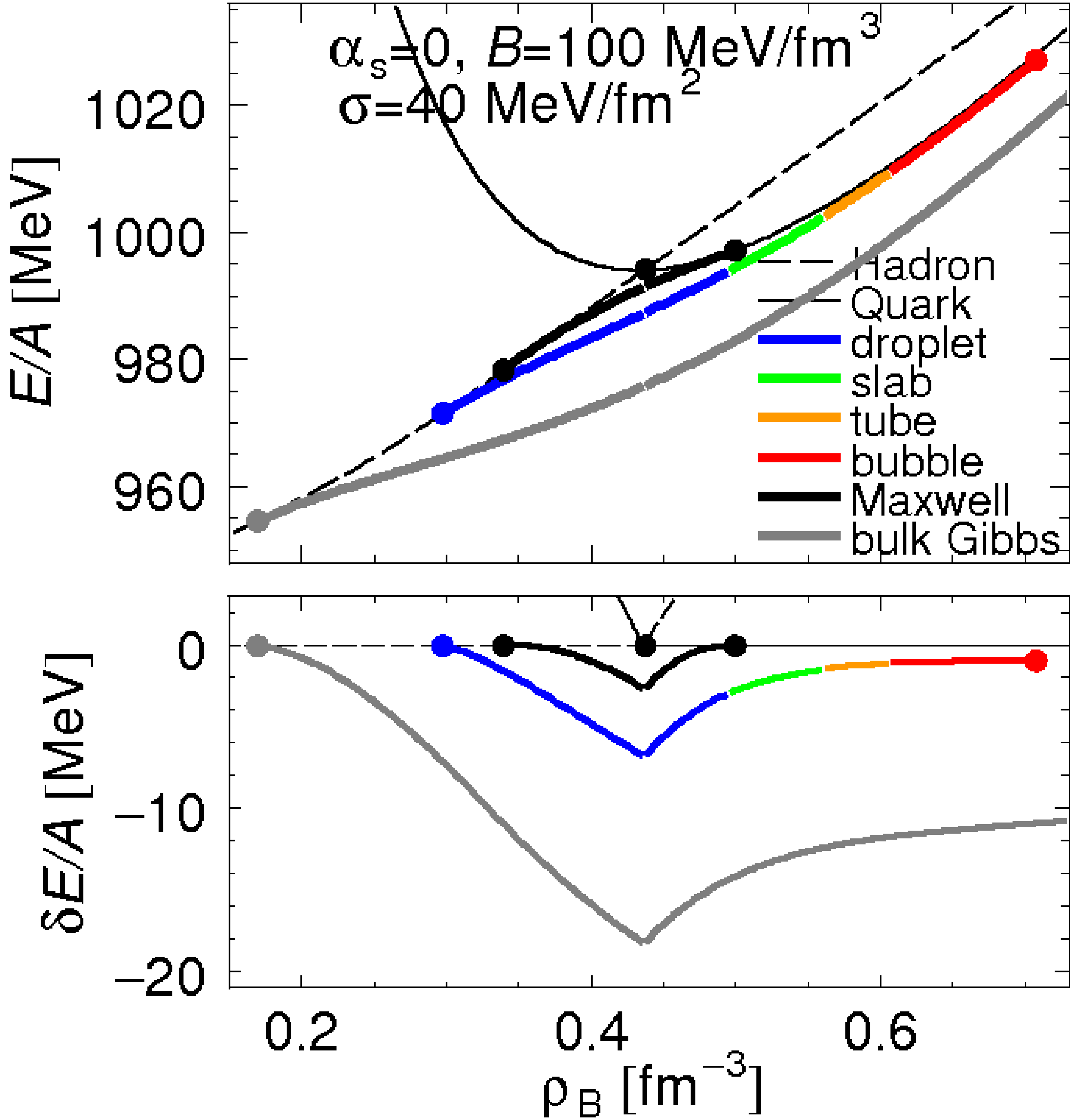
Quark phase is negatively charged.

→ *u* quarks are attracted and *ds* quarks repelled.

Same thing happens to *p* in the hadron phase.

EOS of matter

Full calculation is close to the Maxwell construction (local charge neutral).
Far from the bulk Gibbs calculation (neglects the surface and Coulomb).



Particle fraction

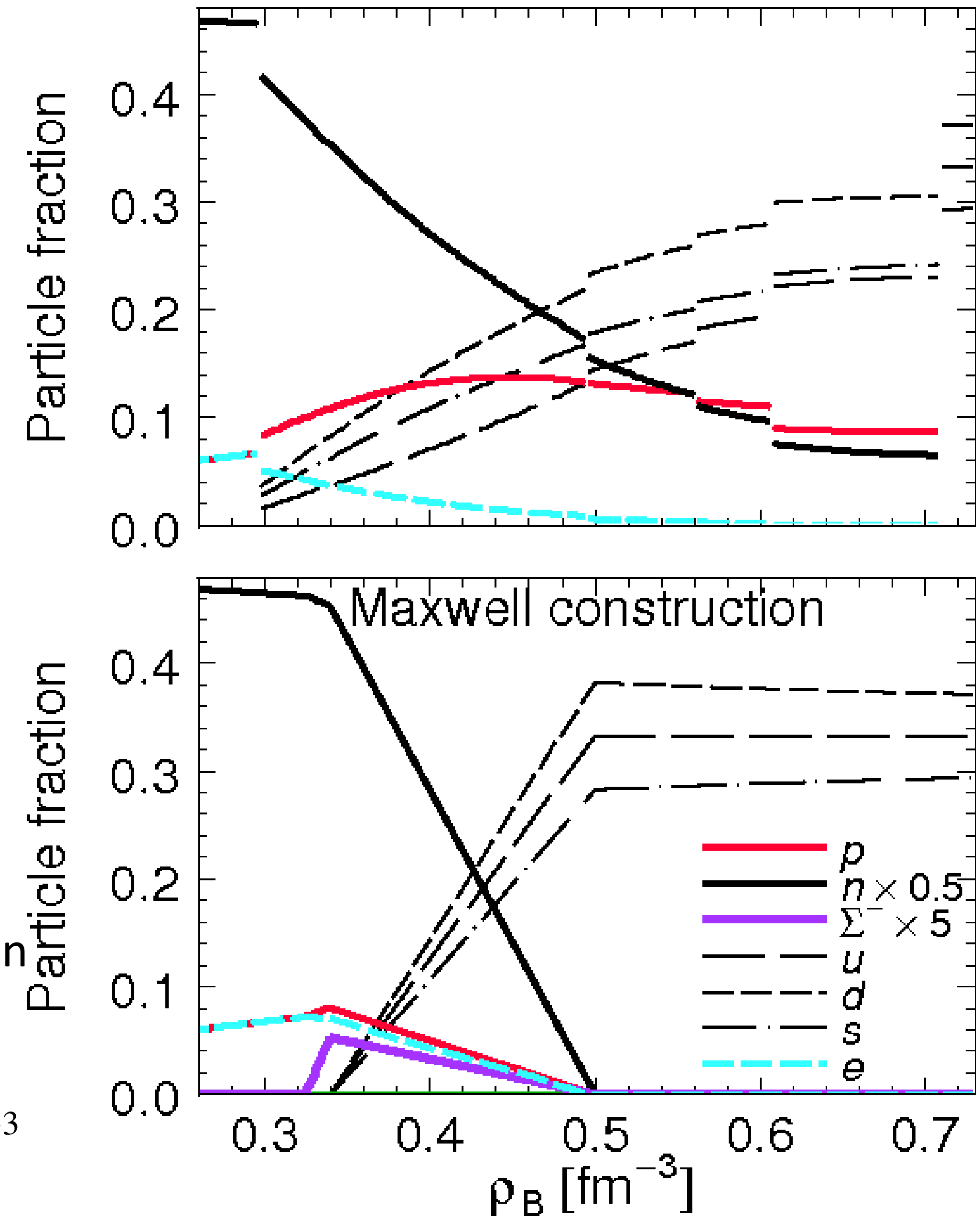
Though the EOS of full calc is close to the Maxwell constrctn, the particle fraction is much different.

Hyperon does not appear.

Lack of the charge-neutrality condition in each phase suppresses hyperons.

Neutral matter → ρ_{th}=0.34 fm^{-3}

Non neutral matter → ρ_{th}=1.15 fm^{-3}



Structure of compact stars

Solve TOV eq.

$$\frac{dP}{dr} = -\rho \frac{Gm}{r^2}\left(1+\frac{4\pi r^3 P}{m}\right)\left(1+\frac{P}{\rho}\right)\left(1-\frac{2Gm}{r}\right)^{-1}$$

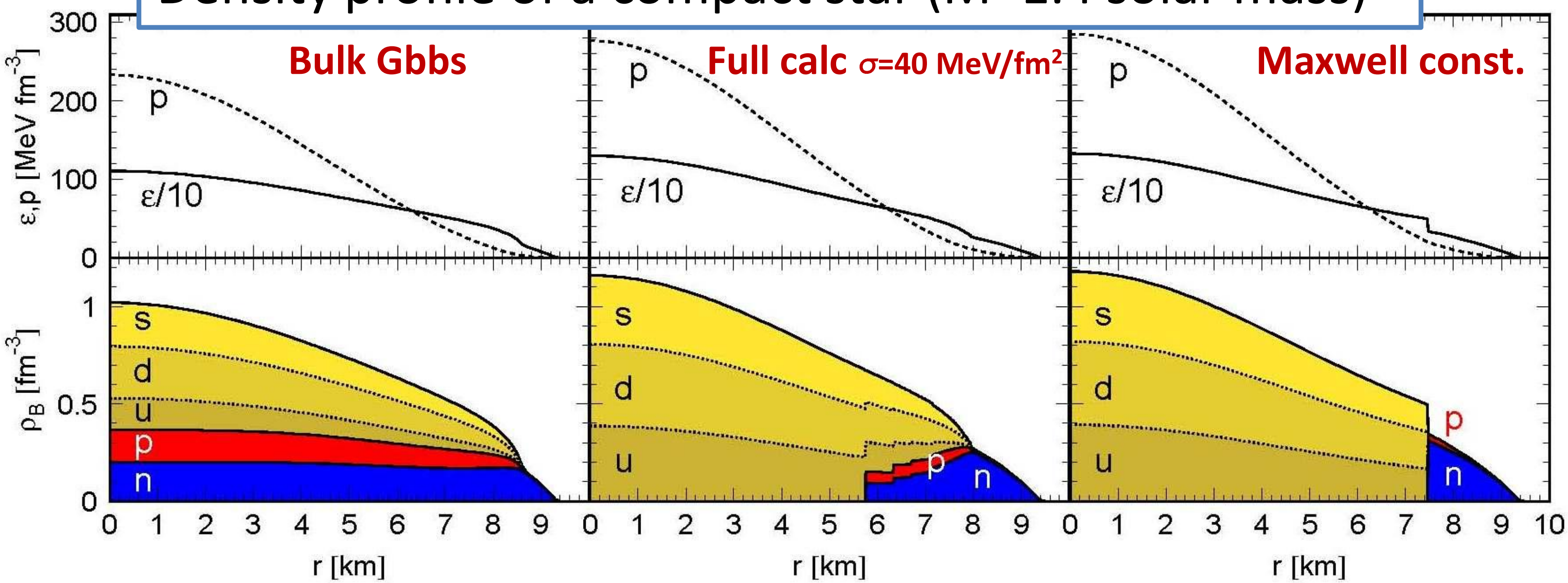
$P = P(\rho)$ Pressure (input of TOV eq.)

$\rho = \rho(r)$ Density at r

$m = m(r) = \int_0^r 4\pi s^2 \rho(s) ds$ Mass inside r

$M = m(R), \quad R = R(\rho \approx 0)$ Total mass and Radius

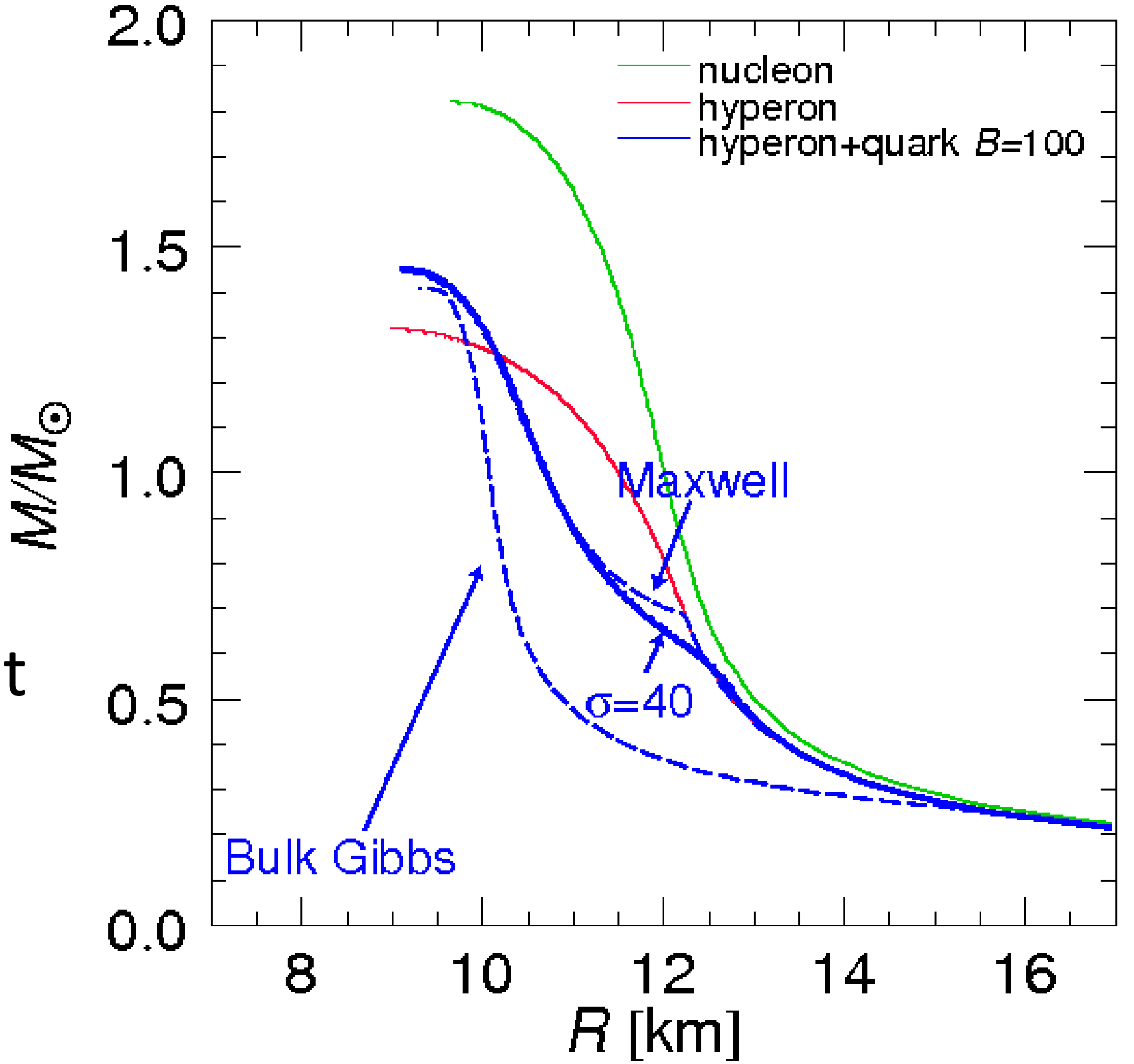
Density profile of a compact star (M=1.4 solar mass)



Mass-Radius relation of compact stars

Full calc yields the neutron star mass very close to that of the Maxwell constr.

The maximum mass are not very different for three cases.



Summary

- We have studied ``Pasta'' structures of Hyperon-quark mixed phase by means of BHF and MIT bag model.
- Resulant EOS of mixed phase is close to that of the Maxwell constrctn instead of a bulk Gibbs calc.
- The mass-radius relation of a compact star is also close to the Maxwell constrctn case.
- But the particle fraction and the inner structure is quite different. Hyperons are strongly suppressed.

  → important for thermal property and ν opacity.