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Relativistic Three-Body Scattering in First Order Faddeev Formulation

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Exploring Nuclear Matter - Quarks to Stars

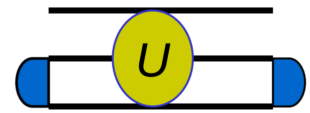
Relativistic Three-Body Problem

- Context: **Poincarė Invariant Quantum Mechanics**
  - Poincarė invariance is exact symmetry, realized by a unitary representation of the Poincarė group on a few-particle Hilbert space
  - Instant form
  - Faddeev equations same operator form but different ingredients
- **Kinematics**
  - Lorentz transformations between frames
- **Dynamics**
  - Bakamjian-Thomas Scheme: Mass Operator $M=M_0+V$
  - Interaction embedded in 3-body space $V \equiv \sqrt{M^2+q^2} - \sqrt{M_0^2+q^2}$

Three-Body Scattering

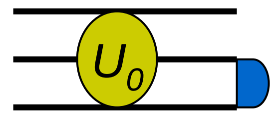
- Transition operator for elastic scattering

$$U = PG_0^{-1} + PT$$



- Transition operator for breakup scattering

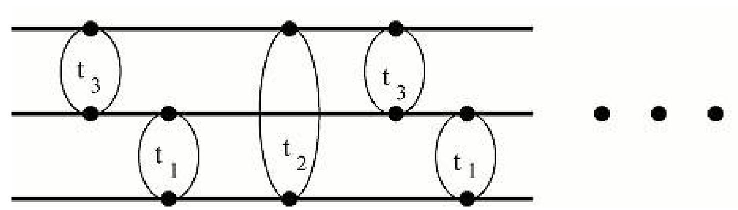
$$U_0 = (1 + P)T$$



- Faddeev equation (Multiple Scattering Series)

$$T = tP + tG_0PtP + \cdots$$

1st Order in tP



$t = v + vg_0t$ =: NN t-matrix

$P = P_{12}P_{23} + P_{13}P_{23} \equiv$ Permutation Operator

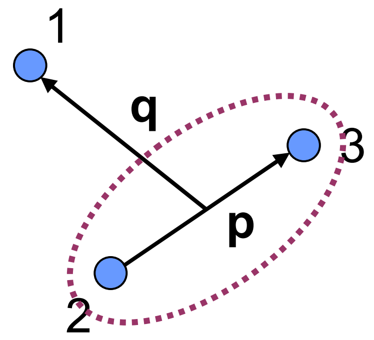
Kinematic Relativistic Effects:

- Lorentz transformation Lab $\rightarrow$ c.m. frame (3-body)
- Phase space factors in cross sections
- Poincarė-Jacobi momenta
- Permutations for identical particles

Kinematics: Poincaré-Jacobi momenta

- **Nonrelativistic (Galilei)**

$$\mathrm{p} = \frac{1}{2}(\mathrm{k}_2 - \mathrm{k}_3)$$
$$\mathrm{q} = \frac{2}{3}(\mathrm{k}_1 - \frac{1}{2}(\mathrm{k}_2 + \mathrm{k}_3))$$



- **Relativistic (Lorentz)**

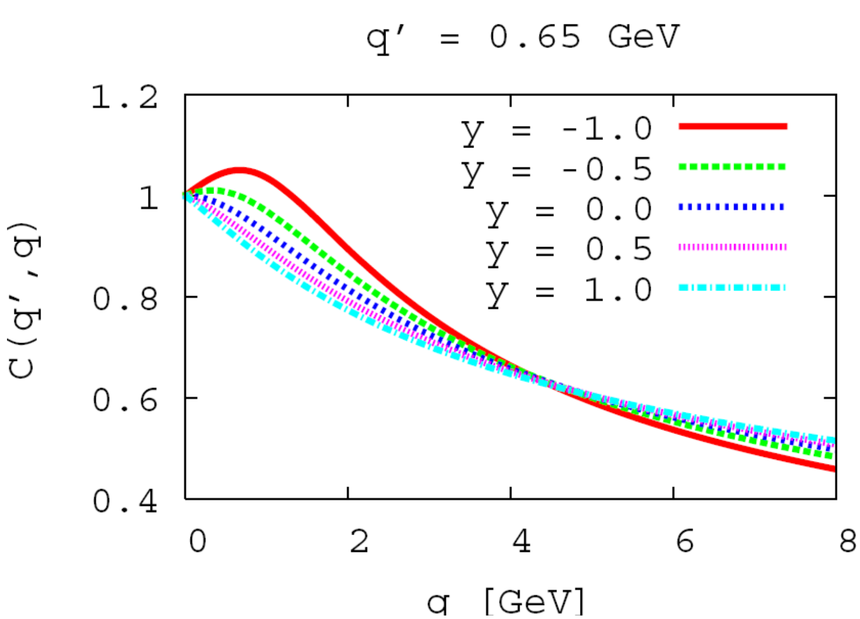
$$\mathrm{p} = \frac{1}{2}(\mathrm{k}_2 - \mathrm{k}_3) + \frac{\mathrm{k}_2 + \mathrm{k}_3}{2m_{23}}\left(\frac{(\mathrm{k}_2 - \mathrm{k}_3)\cdot(\mathrm{k}_2 + \mathrm{k}_3)}{(E_2 + E_3) + m_{23}} - (E_2 - E_3)\right)$$
$$\mathrm{q} = \mathrm{k}_1 + \frac{\mathrm{K}}{M}\left(\frac{\mathrm{k}_1 \cdot \mathrm{K}}{E + M} - E_1\right)$$

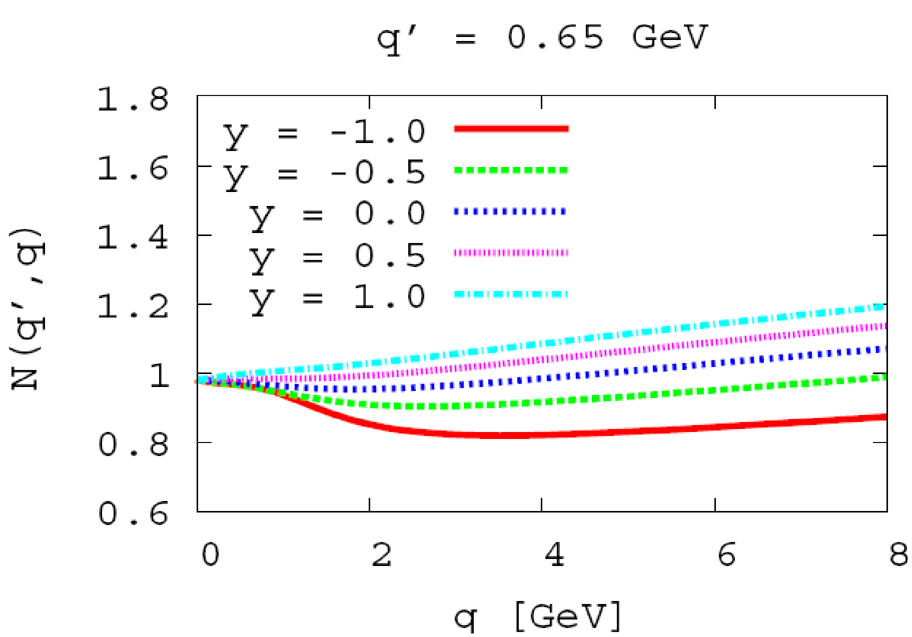
$$\left|\mathrm{k}_1\mathrm{k}_2\mathrm{k}_3\right\rangle = \left|\frac{\partial(K\mathrm{pq})}{\partial(\mathrm{k}_2\mathrm{k}_3)}\right|^{1/2}\left|\mathrm{Kpq}\right\rangle \neq 0$$

$$E = E_1 + E_2 + E_3$$
$$\mathrm{K} = \mathrm{k}_1 + \mathrm{k}_2 + \mathrm{k}_3$$
$$M = \sqrt{E^2 - \mathrm{K}^2}$$
$$m_{23} = \sqrt{(E_2 + E_3)^2 - (\mathrm{k}_2 + \mathrm{k}_3)^2}$$

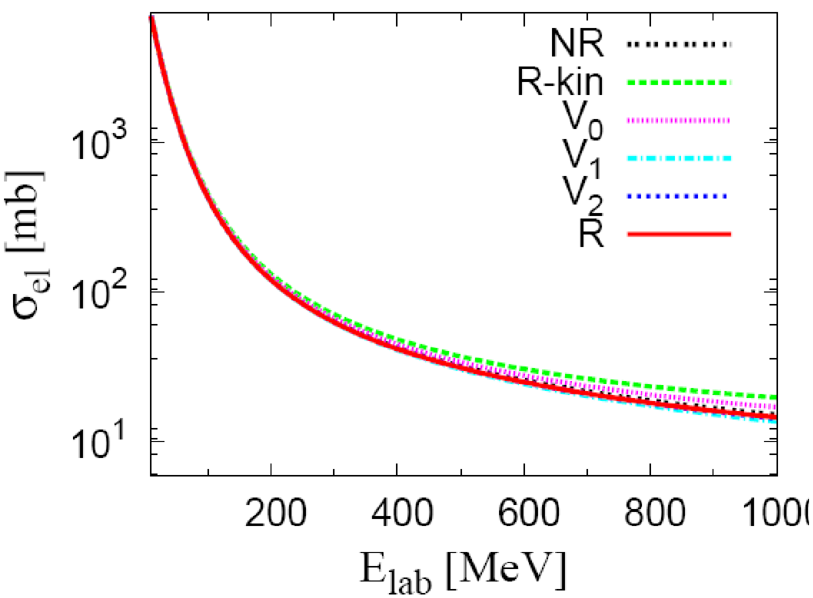
Permutation Operator: $P=P_{12}P_{23}+P_{13}P_{23}$

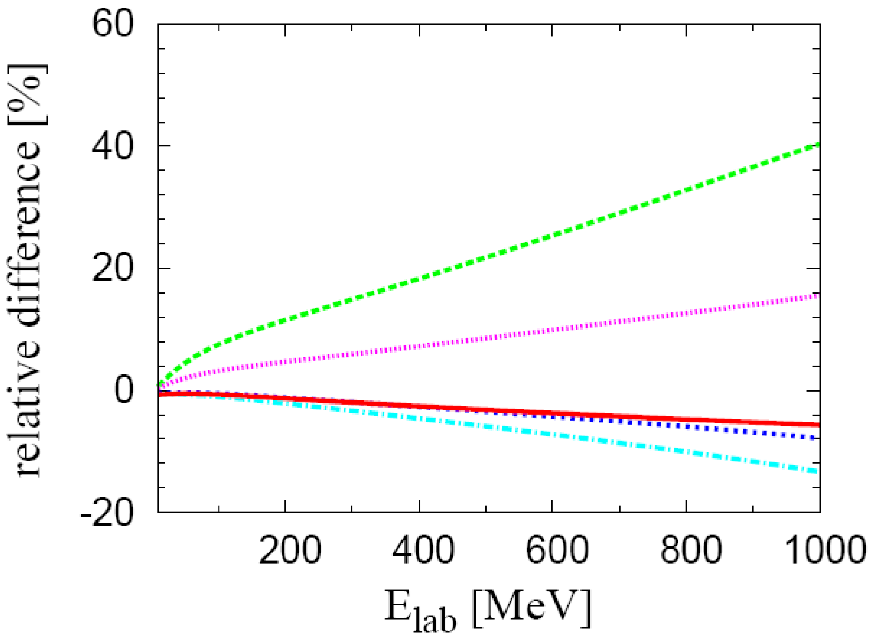
$$
\begin{aligned}
{}_1\langle \mathbf{p}'\mathbf{q}'|P|\mathbf{p}''\mathbf{q}''\rangle_1 &= {}_1\langle \mathbf{p}'\mathbf{q}'|\mathbf{p}''\mathbf{q}''\rangle_2 + {}_1\langle \mathbf{p}'\mathbf{q}'|\mathbf{p}''\mathbf{q}''\rangle_3 \\
&= \hat{N}(\mathbf{q}',\mathbf{q}'')\Big[\delta\left(\mathbf{p}'-\mathbf{q}''-\frac{1}{2}\mathbf{q}'\underline{C(\mathbf{q}'',\mathbf{q}')}\right)\delta\left(\mathbf{p}''+\mathbf{q}'+\frac{1}{2}\mathbf{q}''\underline{C(\mathbf{q}',\mathbf{q}'')}\right) \\
&\quad +\delta\left(\mathbf{p}'+\mathbf{q}''+\frac{1}{2}\mathbf{q}'\underline{C(\mathbf{q}'',\mathbf{q}')}\right)\delta\left(\mathbf{p}''-\mathbf{q}'-\frac{1}{2}\mathbf{q}''\underline{C(\mathbf{q}',\mathbf{q}'')}\right)\Big]
\end{aligned}
$$





Total Cross Section for Elastic Scattering





Relative difference to non-relativistic calculation

Dynamics: Quantum Mechanics

Galilei Invariant: $H = \frac{K^2}{2M_g} + h \quad ; \quad h = h_0 + V_{NR}$

Poincaré Invariant: $H = \sqrt{K^2 + M^2} \quad ; \quad M = M_0 + V_{12} + V_{23} + V_{31}$

$$V_{ij} = M_{ij} - M_0 = \sqrt{\left(m_{0,ij} + v_{ij}\right)^2 + q_k^2} - \sqrt{m_{0,ij}^2 + q_k^2}$$

Two-body interaction embedded in the 3-particle Hilbert space

$$m_{0,ij} = \sqrt{m_i^2 + p_{ij}^2} + \sqrt{m_j^2 + p_{ij}^2}$$

$$M_0 = \sqrt{m_{0,ij}^2 + q_k^2} + \sqrt{m_k^2 + q_k^2}$$

Two-Body Input: T1-operator embedded in 3-body system

$$T_1(\mathrm{p}',\mathrm{p};\mathrm{q}) = V(\mathrm{p}',\mathrm{p};\mathrm{q}) + \int d^3k'' \frac{V(\mathrm{p}',\mathrm{k}'';\mathrm{q})\, T_1(\mathrm{k}'',\mathrm{p};\mathrm{q})}{\sqrt{(2E(p'))^2+q^2} - \sqrt{(2E(k''))^2+q^2} + i\varepsilon}$$

Do not solve!

- Obtain fully off-shell matrix elements T_1(k,k',q) from half shell transition matrix elements by

  Solving a 1st resolvent type equation:

$$T_1(q) = T_1(q') + T_1(q)\,[g_0(q) - g_0(q')]\,T_1(q')$$

- For every single off-shell momentum point
- Proposed in
  - Keister & Polyzou, PRC 73, 014005 (2006)
- Carried out for the first time here [nucl-th/0702005]

Exact Boost

Obtain embedded 2N t-matrix $T_1(k,k',z')$:

$$\begin{aligned} \langle \mathbf{k}|T_1(\mathbf{q};z')|\mathbf{k}'\rangle &= \langle \mathbf{k}|V(\mathbf{q})|\mathbf{p}'^{(-)}\rangle \\ &= \frac{2(E_{k'}+E_k)}{\sqrt{4E_{k'}^2+\mathbf{q}^2}+\sqrt{4E_k^2+\mathbf{q}^2}}\, t(\mathbf{k},\mathbf{k}';2E_{k'}) \end{aligned}$$

$$t(\mathbf{k},\mathbf{k}';2E_{k'}) = v(\mathbf{k},\mathbf{k}') + \int d\mathbf{k}'' \frac{v(\mathbf{k},\mathbf{k}'')t(\mathbf{k}'',\mathbf{k}';2E_{k'})}{E_{k'} - 2\sqrt{m^2+k''^2} + i\epsilon}$$

Solution of the relativistic 2N LS equation with 2-body potential

Approximations to the "boosted" potential

$$V = \sqrt{(2\sqrt{m^2+\mathbf{p}^2}+v)^2+\mathbf{q}^2} - \sqrt{4(m^2+\mathbf{p}^2)+\mathbf{q}^2}$$

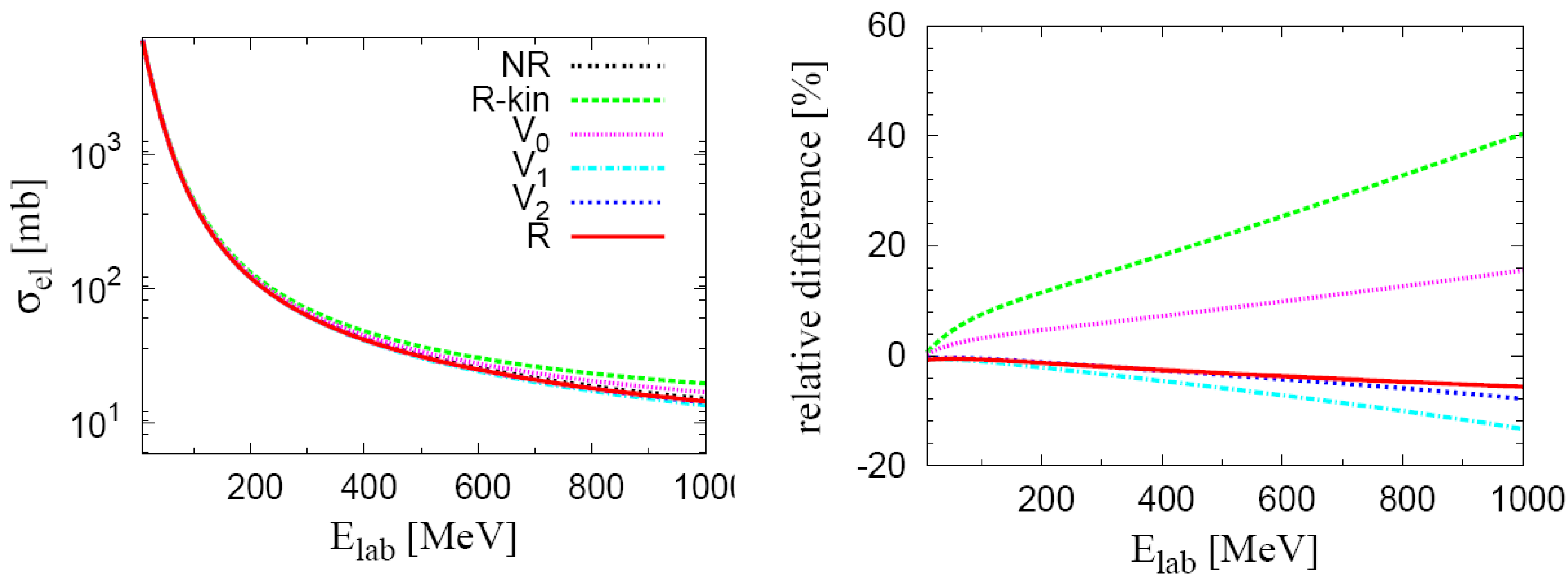
$$
\begin{aligned}
V_0(\mathbf{p},\mathbf{p}',\mathbf{q}) &= v(\mathbf{p},\mathbf{p}') \\
V_1(\mathbf{p},\mathbf{p}',\mathbf{q}) &= v(\mathbf{p},\mathbf{p}')\left[1-\frac{\mathbf{q}^2}{8m^2}\right] \\
V_2(\mathbf{p},\mathbf{p}',\mathbf{q}) &= v(\mathbf{p},\mathbf{p}')\left[1-\frac{\mathbf{q}^2}{8E(\mathbf{p})E(\mathbf{p}')}\right]
\end{aligned}
$$

relativistic interaction in the c.m. frame

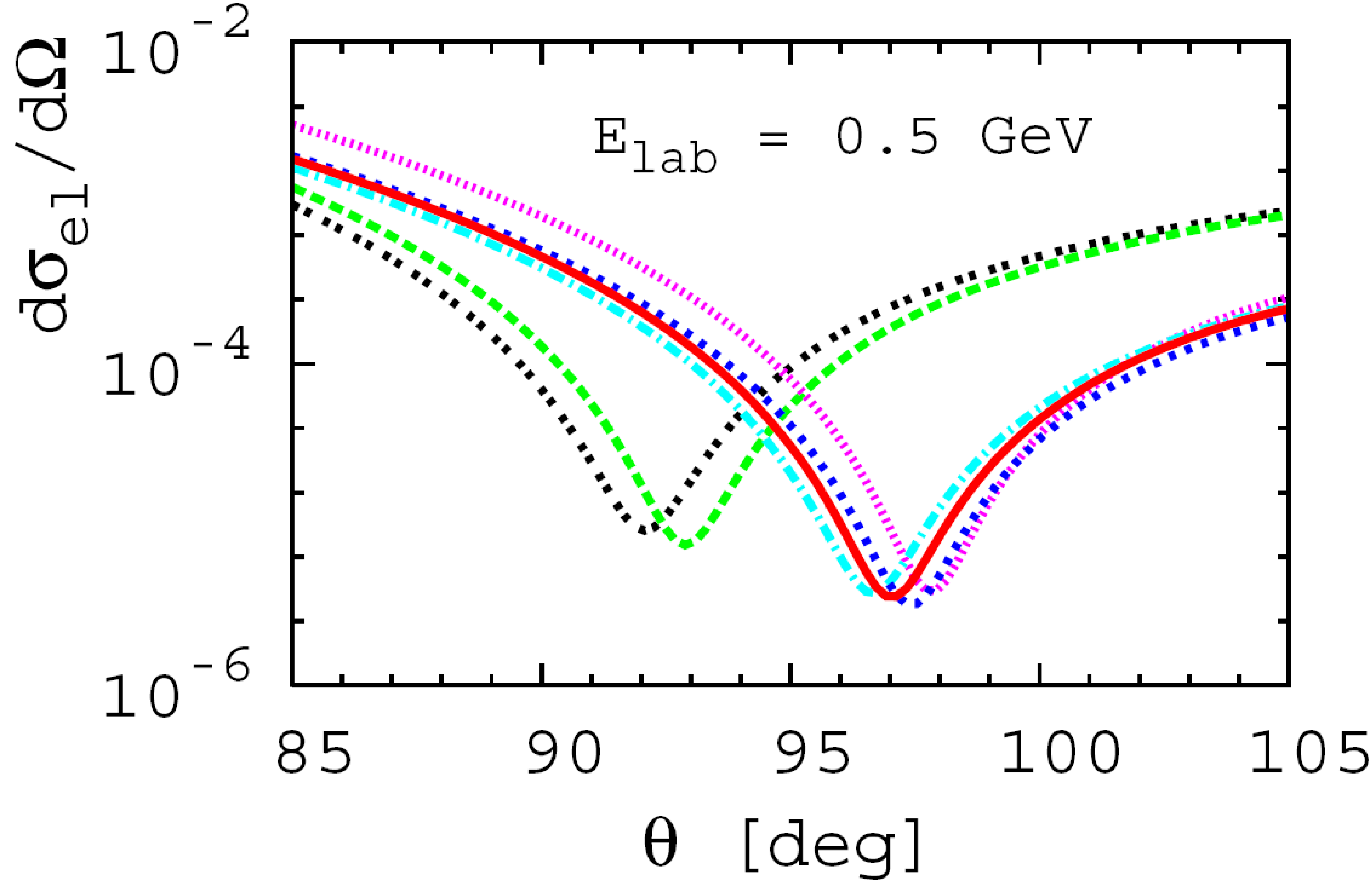
Remark to calculations:

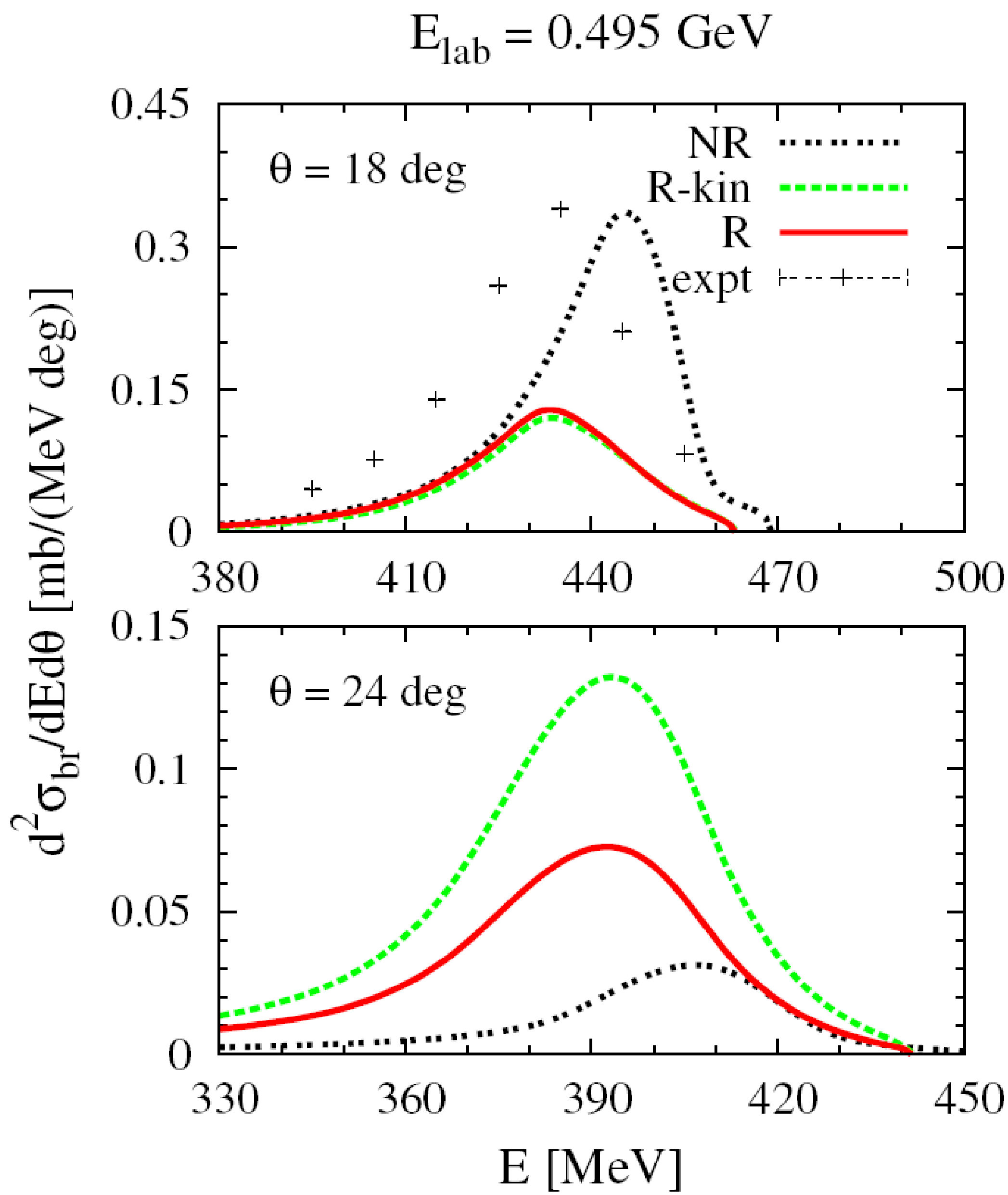
The relativistic potential v(p,p') is phase-shift equivalent to the nonrelativistic potential

Total Cross Section for Elastic Scattering



Elastic Scattering: Differential Cross Section





Inclusive Breakup Scattering @ Elab=495 MeV

Relativistic Faddeev Calculations in 1st Order in t

- The relativistic Faddeev equation is solved consistently in 1st order in two-body interaction up to 1 GeV projectile energy
  - 1st Order = Born term determines kernel of Faddeev Eq.
  - No partial wave decomposition

- **Exact calculation of the two-body interaction embedded in the three-particle Hilbert space via first resolvent equations**
  - Calculations at intermediate energy show that relativistic effects are quite visible.
- Expansions in **q**/m and **p**/m give quite good (~5%) result up to ~ 500 MeV.
  - Discrepancies get larger with increasing energies.

In Progress: Full Solution of Relativistic Faddeev Equation