The Tensor Part of the Skyrme Energy Density Functional

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Introduction

- The tensor force is an essential part of the NN interaction (^{3}S-D$_1$ phase shifts, deuteron quadrupole moment. . .).
- Early studies in a mean-field framework:
    - T.H.R. Skyrme included it (in analytial expresions. . .) from the beginning – considered standard s.o. too simple
    - F. Stancu, H. Flocard, D.M. Brink, PLB (1977)
    - K. F. Liu et al., NPA (1991)
- Recent attempts at adding a tensor term to mean-field/density functional models:
    - Gogny: T. Otsuka, T. Matsuo and D. Abe, PRL (2005)
    - RHF: W. H. Long, N. V. Giai and J. Meng, PLB (2006)
    - Skyrme EDF:
        - Perturbative studies:

          G. Colò, H. Sagawa, S. Fracasso and P. Bortignon, PLB (2007);

          D. M. Brink and F. Stancu, nucl-th/0702065 (2007)
        - Refit:

          B. A. Brown, T. Duguet, T. Otsuka, D. Abe and T. Suzuki, PRC (2006)

Tensor terms in the nuclear density functional

- V_{Skyrme}+ Zero-range tensor force :

$$
\begin{aligned}
v^{\text{t}}(\vec{r}) \quad = \quad & \tfrac{1}{2}\, t_e \left\{ \left[3\,(\sigma_1 \cdot \vec{k}')\,(\sigma_2 \cdot \vec{k}') - (\sigma_1 \cdot \sigma_2)\, \vec{k}'^2 \right] \delta(\vec{r}) + \text{h.c.} \right\} \\
& + t_o \left[3\,(\sigma_1 \cdot \vec{k}')\, \delta(\vec{r})\, (\sigma_2 \cdot \vec{k}) - (\sigma_1 \cdot \sigma_2)\, \vec{k}' \cdot \delta(\vec{r})\, \vec{k} \right]
\end{aligned}
$$

- Potential energy density in a spherical even-even nucleus:

$$
\begin{aligned}
\mathscr{H}^{\text{Skyrme}} = \sum_{t=0,1} \Big\{ \quad & C_t^{\rho}[\rho_0]\, \rho_t^2 + C_t^{\Delta\rho}\, \rho_t \Delta \rho_t + C_t^{\tau}\, \rho_t \tau_t \\
& + \tfrac{1}{2}\, C_t^{J}\, \vec{J}_t^2 + C_t^{\nabla \cdot J}\, \rho_t \nabla \cdot \vec{J}_t \quad \Big\}
\end{aligned}
$$

- Spin-orbit current density (radial)

$$
J_q(r) = \frac{1}{4\pi r^3} \sum_{n,j,\ell} (2j+1)\, v_{nj\ell}^2 \left[j(j+1) - \ell(\ell+1) - \tfrac{3}{4} \right] \psi_{nj\ell}^2(r)
$$

- States with $j_> = \ell + \frac{1}{2}$ and $j_< = \ell - \frac{1}{2}$: $\sim$ opposite contributions
  - ➡ Spin saturation ($J_q = 0$) when s.o. partners both filled
- "Tensor terms": $C_t^J = A_t^J$ (central) $+B_t^J$ (tensor)

$$
\mathscr{H}^{\text{t}} = C_0^J\, \vec{J}_0^2 + C_1^J\, \vec{J}_1^2 = \tfrac{1}{2}\, \alpha\, (\vec{J}_n^2 + \vec{J}_p^2) + \beta\, \vec{J}_n \cdot \vec{J}_p
$$

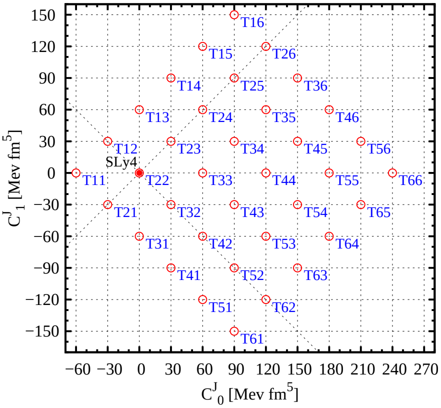
Parameterizations

- Fit procedure: ∼SLy4, E. Chabanat et al., NPA (1998)
  - Properties of SNM and PNM constrained (E/A, ρ_{sat} / EOS) or fixed (a_τ, K_∞)
  - Masses and charge radii of ^{40,48}Ca, ^{56}Ni, ^{90}Zr, ^{132}Sn, ^{208}Pb, mass of ^{100}Sn
  - Spin-orbit splitting of $\nu 3p$ state in ^{208}Pb constrained

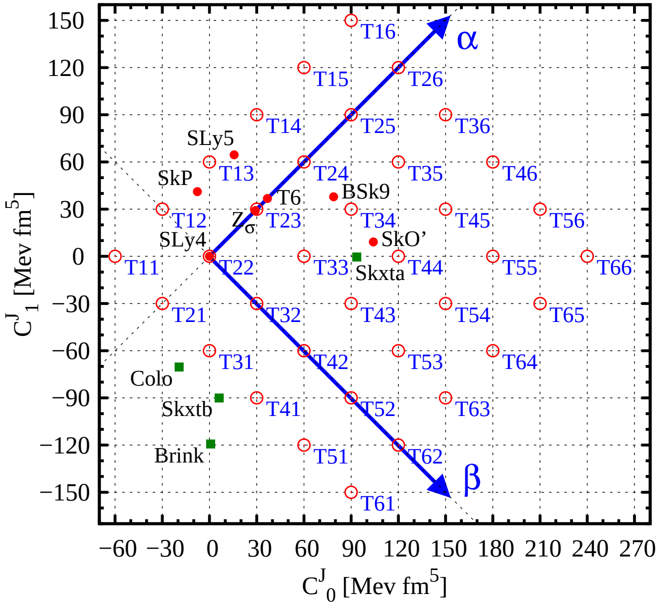
➡ 36 parameter sets TIJ with

$$\alpha = 60\,(J-2)\ \text{MeV fm}^5$$
$$\beta = 60\,(I-2)\ \text{MeV fm}^5$$

- T22: No tensor coupling (∼SLy4)
- Spin-orbit parameter W_0: 103.7 to 195.3 MeV fm^5)

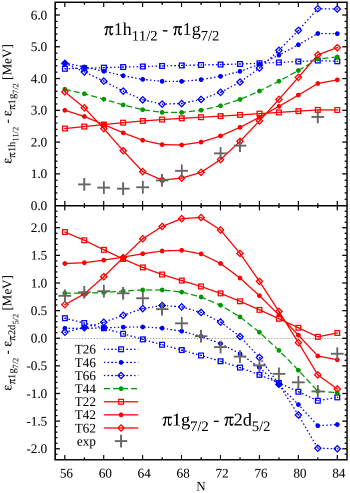


Parameterizations

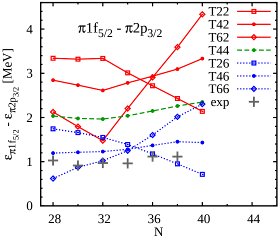


Evolution of single-particle levels: np coupling (β)

- S.o. field: $W_p(r) = \frac{W_0}{2}\left(2\nabla\rho_p + \nabla\rho_n\right) + \alpha\, J_p + \beta\, J_n$
- Difference of two proton s.p.e.'s in an isotopic magic series



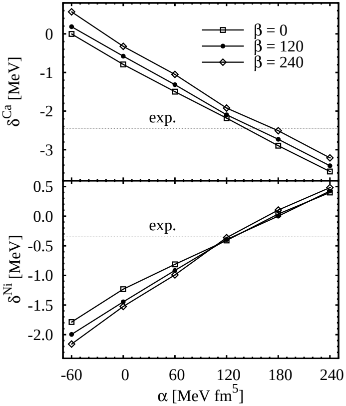
- ⇐ Sn: filling $\nu\, 1h_{11/2}$
- ⇓ Ni: filling $\nu\, 1f_{5/2}$
- ➡ α (color): global shift
- ➡ $\beta \sim 120$ (T4J) matches slope and curvature of data best
- ✘ Averge relative position of levels "off"



Evolution of single-particle levels: nn coupling (α)

- S.o. field: $W_n(r) = \frac{W_0}{2}\left(2\nabla\rho_n + \nabla\rho_p\right) + \alpha\, J_n + \beta\, J_p$
- Calcium isotopes: $1d_{3/2}$, $2s_{1/2}$ separated by 2.4 MeV in ^{40}Ca, almost degenerate in ^{48}Ca
- Double difference between two neutron levels in two isotopes

$$\delta^{\mathrm{Ca}} = \left(\varepsilon^{^{48}\mathrm{Ca}}_{1d_{3/2}} - \varepsilon^{^{48}\mathrm{Ca}}_{2s_{1/2}}\right) - \left(\varepsilon^{^{40}\mathrm{Ca}}_{1d_{3/2}} - \varepsilon^{^{40}\mathrm{Ca}}_{2s_{1/2}}\right)$$

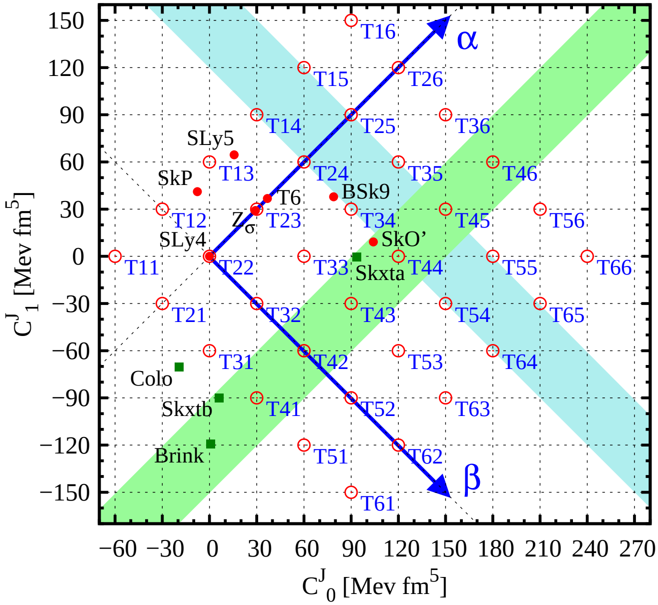


- Nickel isotopes: $1f_{5/2}$, $2p_{1/2}$

$$\delta^{\mathrm{Ni}} = \left(\varepsilon^{^{68}\mathrm{Ni}}_{1f_{5/2}} - \varepsilon^{^{68}\mathrm{Ni}}_{2p_{1/2}}\right) - \left(\varepsilon^{^{56}\mathrm{Ni}}_{1f_{5/2}} - \varepsilon^{^{56}\mathrm{Ni}}_{2p_{1/2}}\right).$$

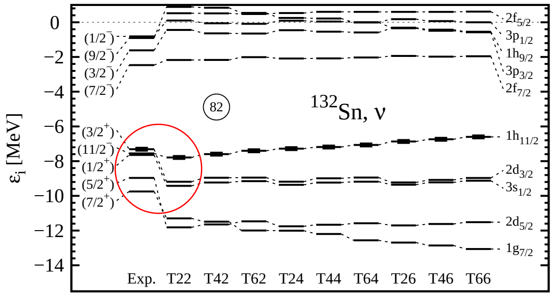
✓ Relative evolution well reproduced for $\alpha = 120 - 180$

Parameterizations



Single-particle spectra of heavy nuclei

- ^{132}Sn: $\nu 1h_{11/2}$ too high. . . Even worse with $\alpha, \beta > 0$
  - Wrong ℓ-dependence of s.o. splittings
- Centroid positions ?
- ^{132}Sn: $\nu 1h$
  - ➨ $m^* < 1$ should only cause a scaling of s.p.e.'s



- ^{208}Pb: $\nu 1i$, $\pi 1h$

✘ Centroid energies too high in theory vs. experiment. Quantity related to the central potential

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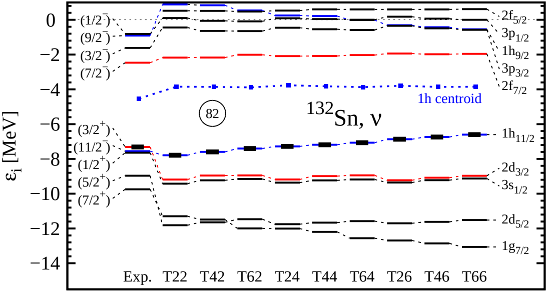

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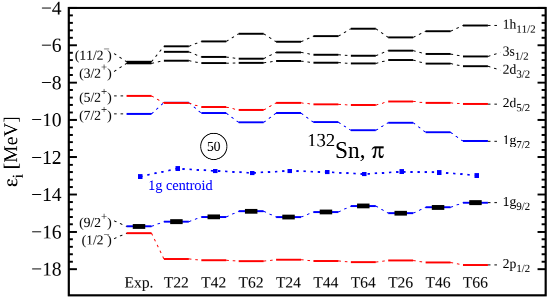

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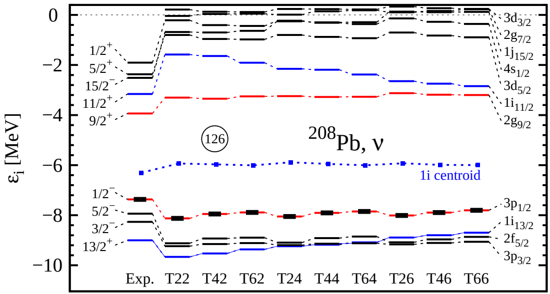

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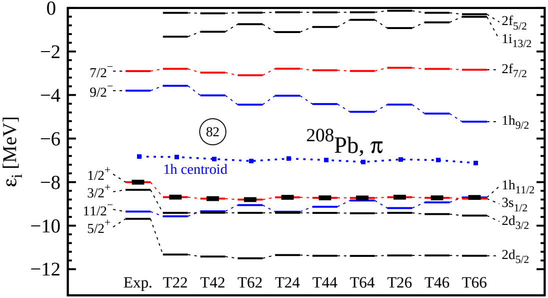

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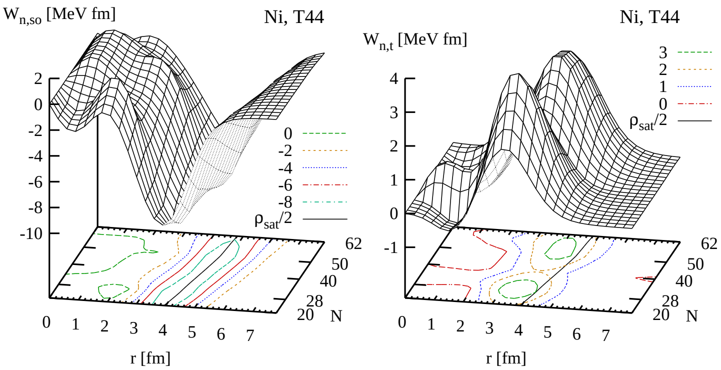
Conclusions

- Strong rearrangement of the spin-orbit term: full refit mostly needed
- S.o. and tensor contributions to s.o. field differ in localization
  - State-dependent effects (hig-ℓ intruder states/low-ℓ states) in s.o. splittings – see spectra of ^{132}Sn, ^{208}Pb
  - Additionally, centroid energies of nodeless high-ℓ states are wrong
- Evolution of s.p. states in isotopic series calls for $\alpha \sim \beta \sim 120$ MeV fm^5.
  - ✘ Tends to make problems in ^{132}Sn & ^{208}Pb worse

➡ Extensions of central/spin-orbit terms needed to correct ℓ-dependence of splittings and centroid energies.

T. L., M. Bender, K. Bennaceur, T. Duguet and J. Meyer,
PRC (2007), arxiv:0704.0731

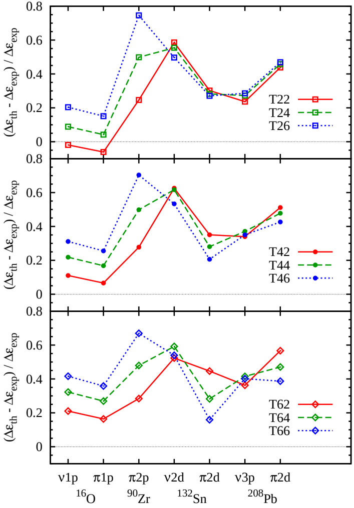
Spin-orbit fields



$$W_n(r) = \frac{W_0}{2}\left(2\nabla\rho_n + \nabla\rho_p\right) + \alpha\, J_n + \beta\, J_p$$

- Tensor contribution varies rapidly with filling of spin-orbit partner states
- Spin-orbit and tensor contributions to spin-orbit field are peaked at slightly different positions w.r.t. the nuclear surface
- ➡ The tensor force alters shape as well as magnitude of the spin-orbit field

Spin-orbit splittings



- Splittings of p, d levels ($n \geq 2$) larger than empirical values.
- Splittings of f, g, h, i ($n = 1$) levels underestimated
- Shape of s.o. field $\Rightarrow$ state-dependence of splittings

➨ Nodeless states are the most affected by tensor terms

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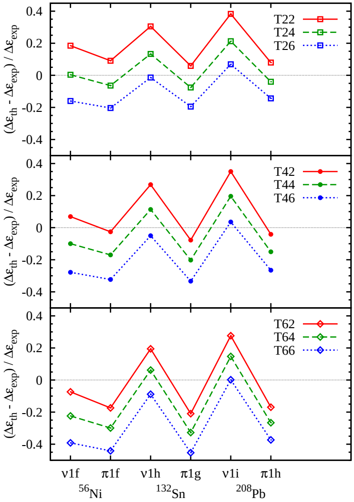

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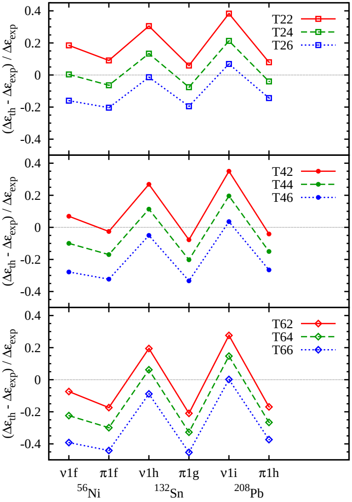

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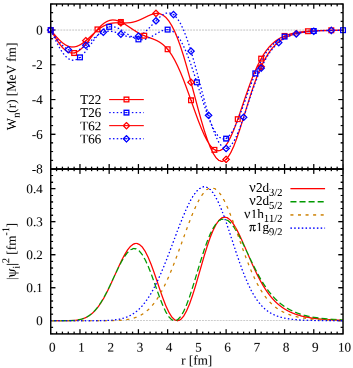

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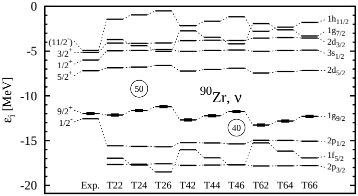
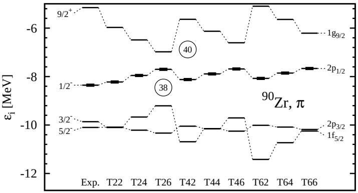
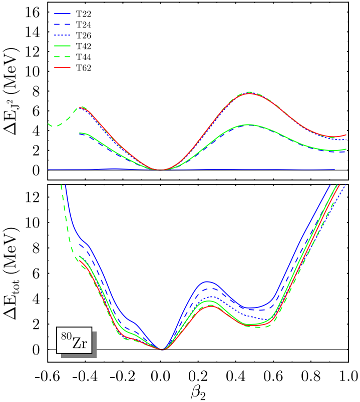
Deformation: the Zirconium chain

- ^{80}Zr: well deformed, ^{90}Zr: weakly magic, ^{100}Zr: shape coex.







- ^{80}Zr: Weakly sensitive; deformed minimum lowered with $\alpha, \beta > 0$
- ^{90}Zr: less magic with $\alpha > 0$
- ^{100}Zr: with $\beta > 0$, $\alpha > 0$ needed to preserve shape coexistence.

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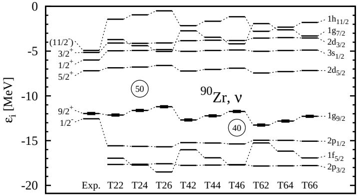

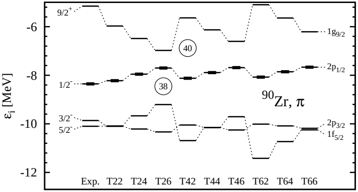

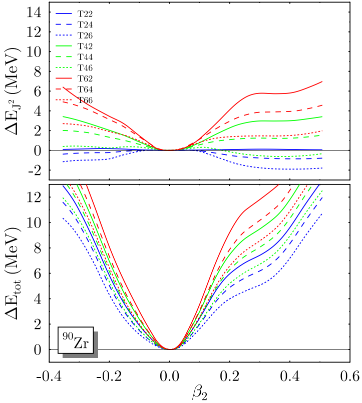

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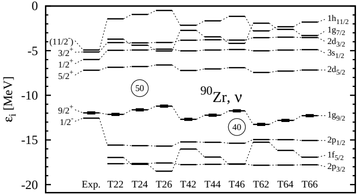

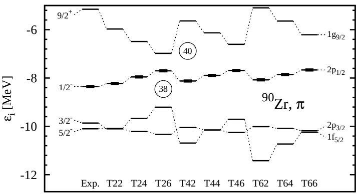

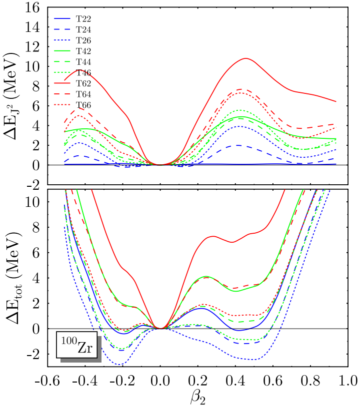

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Spin saturation: light vs. heavy