$S = -1, -2$ baryon-baryon interactions in chiral effective field theory

Henk Polinder

IKP, Forschungszentrum GmbH, Jülich, Germany

23rd International Nuclear Physics Conference (INPC2007)
June 3-8, 2007, Tokyo, Japan

Outline

1. Introduction
2. Strangeness -1 YN in chiral effective field theory
3. Strangeness -2 YY and ΞN in chiral effective field theory
4. Summary and outlook

Chiral effective field theory

No investigations for YY and ΞN, only few investigations for YN so far:

- S.R. Beane, P.F. Bedaque, A. Parreño, M.J. Savage, NPA747 (2005) 55
- C.K. Korpa, A.E.L. Dieperink, R.G.E. Timmermans, PRC 65 (2001) 015208
  (pion-less theory; Kaplan-Savage-Wise resummation scheme)

We follow the scheme of Epelbaum et al., succesfully applied to NN at N3LO:

- E. Epelbaum, W. Glöckle, Ulf-G. Meißner, NPA747 (2005) 362
  (Weinberg power counting applied to potential, potential consists of contact terms and pion exchanges, regularized LS equation solved)

and have applied it to $S = -1$ YN, starting at LO:

- Henk Polinder, Johann Haidenbauer, Ulf-G. Meißner, NPA779 (2006) 244

and have extended this work to $S = -2$ YY and ΞN, also at LO:

- H. Polinder, J. Haidenbauer, U.-G. Meißner, arXiv:0705.3753 [nucl-th]

Leading order (LO) contact term for NN

Contact term for the NN interaction:

$$\mathcal{L} = C_i \left(\bar{N}\Gamma_i N\right)\left(\bar{N}\Gamma_i N\right)$$

$$\Gamma_1 = 1\,,\ \Gamma_2 = \gamma^\mu\,,\ \Gamma_3 = \sigma^{\mu\nu}\,,\ \Gamma_4 = \gamma^\mu\gamma_5\,,\ \Gamma_5 = \gamma_5$$

Considering the large components of the nucleon spinors only, the LO contact term becomes

$$\mathcal{L} = -\frac{1}{2}C_S\left(\varphi_N^\dagger\varphi_N\right)\left(\varphi_N^\dagger\varphi_N\right) - \frac{1}{2}C_T\left(\varphi_N^\dagger\boldsymbol{\sigma}\varphi_N\right)\left(\varphi_N^\dagger\boldsymbol{\sigma}\varphi_N\right)$$

The LO contact term potential resulting from the interaction Lagrangian:

$$V^{(LO)} = C_S + C_T\, \boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$$

C_S and C_T are constants; are determined in a fit to the experimental data.

LO contact terms for YN

Contact terms for the YN interaction:

$$\mathcal{L}^1 = C_i^1 \left\langle \bar{B}_a \bar{B}_b \left(\Gamma_i B\right)_b \left(\Gamma_i B\right)_a \right\rangle \ , \quad \mathcal{L}^2 = C_i^2 \left\langle \bar{B}_a \left(\Gamma_i B\right)_a \bar{B}_b \left(\Gamma_i B\right)_b \right\rangle \ ,$$
$$\mathcal{L}^3 = C_i^3 \left\langle \bar{B}_a \left(\Gamma_i B\right)_a \right\rangle \left\langle \bar{B}_b \left(\Gamma_i B\right)_b \right\rangle \ ,$$

a, b denote the Dirac indices of the particles , B is the irreducible octet representation of $\mathrm{SU}(3)_f$:

$$B = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & \Sigma^+ & p \\ \Sigma^- & \frac{-\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} & n \\ -\Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix}$$

e.g. $\mathcal{L}^3 \rightarrow C_i^3 \left\{ 2 \left(\bar{\Lambda}\Gamma_i\Lambda\right) \left(\bar{N}\Gamma_i N\right) + 2 \left(\bar{\mathbf{\Sigma}} \cdot \Gamma_i \mathbf{\Sigma}\right) \left(\bar{N}\Gamma_i N\right) + \left(\bar{N}\Gamma_i N\right) \left(\bar{N}\Gamma_i N\right) \right\}$

Six LO contact terms (C_S^1, C_T^1, C_S^2, C_T^2, C_S^3 and C_T^3) for the BB interactions!

LO contact terms for YN

The six contact terms enter the YN potentials in only 5 different combinations. It is convenient to consider linear combinations:

$$
\begin{aligned}
V^{\Sigma\Sigma;3/2}_{1S0} &= C^{\Sigma\Sigma}_{1S0}, & V^{\Sigma\Sigma;3/2}_{3S1} &= C^{\Sigma\Sigma}_{3S1}, \\
V^{\Lambda\Lambda}_{1S0} &= C^{\Lambda\Lambda}_{1S0}, & V^{\Lambda\Lambda}_{3S1} &= C^{\Lambda\Lambda}_{3S1}, \\
V^{\Lambda\Sigma}_{1S0} &= 3\left(C^{\Lambda\Lambda}_{1S0} - C^{\Sigma\Sigma}_{1S0}\right), & V^{\Lambda\Sigma}_{3S1} &= C^{\Lambda\Sigma}_{3S1}, \\
V^{\Sigma\Sigma;1/2}_{1S0} &= 9C^{\Lambda\Lambda}_{1S0} - 8C^{\Sigma\Sigma}_{1S0}, & V^{\Sigma\Sigma;1/2}_{3S1} &= C^{\Lambda\Lambda}_{3S1}.
\end{aligned}
$$

Search for $C^{\Lambda\Lambda}_{1S0}$, $C^{\Lambda\Lambda}_{3S1}$, $C^{\Sigma\Sigma}_{1S0}$, $C^{\Sigma\Sigma}_{3S1}$, and $C^{\Lambda\Sigma}_{3S1}$ in the fitting procedure
We fit to the low-energy cross sections + inelastic capture ratio at rest (in total 35 data)

We do not consider the *NN* interaction explicitly, since it can not be described well in LO

One-pseudoscalar-meson exchange

LO $\mathrm{SU}(3)_f$-invariant pseudoscalar-meson-baryon interaction Lagrangian:

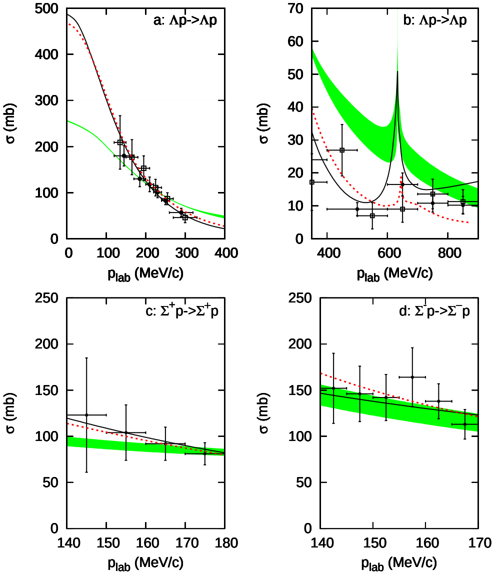
$$\mathcal{L} = -\left\langle \frac{g_A(1-\alpha)}{\sqrt{2}F_\pi}\bar{B}\gamma^\mu\gamma_5\left\{\partial_\mu P, B\right\} + \frac{g_A\alpha}{\sqrt{2}F_\pi}\bar{B}\gamma^\mu\gamma_5\left[\partial_\mu P, B\right]\right\rangle$$

$F_\pi = 92.4$ MeV is the weak pion decay constant, $g_A = F + D \simeq 1.26$ is the axial-vector strength, and for $\alpha = F/(F+D)$ we use the SU(6) value $\alpha = 0.4$

$$P = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & \frac{-\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ -K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix}$$

LO potential: $V^{(LO)} = -f_{B_1B_1'P}f_{B_2B_2'P}\frac{(\boldsymbol{\sigma}_1\cdot\mathbf{k})(\boldsymbol{\sigma}_2\cdot\mathbf{k})}{\mathbf{k}^2+m_P^2}$

YN integrated cross sections



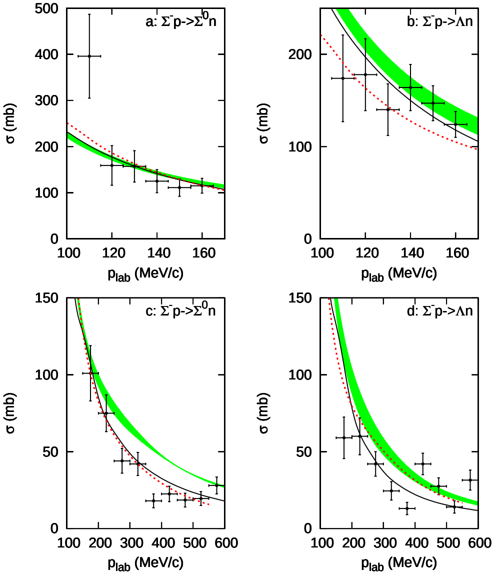
Experimental data in upper right figure not included in fit

EFT '06

Jülich '04

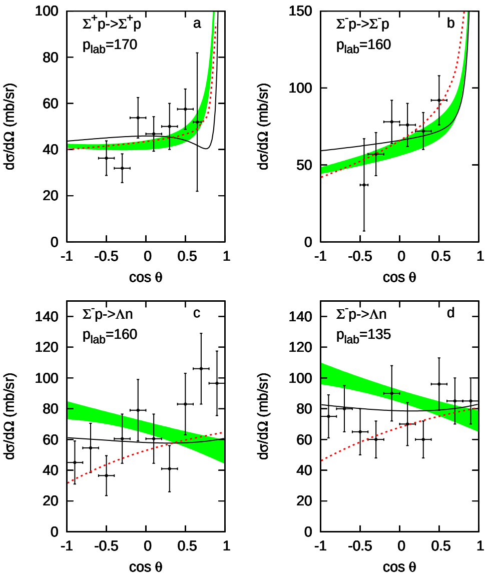
Nijmegen NSC97f

YN integrated cross sections



Experimental data in lower figures not included in fit

YN differential cross sections



Experimental data not included in fit

EFT '06

Jülich '04

Nijmegen NSC97f

YN scattering lengths (fm) and ${}^{3}_{\Lambda}\mathrm{H}$ binding energy

	EFT '06				Jülich '04	NSC97f
Λ [MeV]	550	600	650	700		
$a_s^{\Lambda p}$	-1.90	-1.91	-1.91	-1.91	-2.56	-2.51
$a_t^{\Lambda p}$	-1.22	-1.23	-1.23	-1.23	-1.66	-1.75
$a_s^{\Sigma^+ p}$	-2.24	-2.32	-2.36	-2.29	-4.71	-4.35
$a_t^{\Sigma^+ p}$	0.70	0.65	0.60	0.56	0.29	-0.25
$E_B({}^{3}_{\Lambda}\mathrm{H})$	-2.35	-2.34	-2.34	-2.36	-2.27	-2.30
$\chi^2(35\mathrm{data})$	29.6	28.3	30.3	34.6		

Faddeev calculations for ${}^{3}_{\Lambda}\mathrm{H}$ done by Andreas Nogga, see

- A. Nogga, arXiv:nucl-th/0611081

EFT yields a correctly bound hypertriton: $E_B^{\mathrm{exp}}({}^{3}_{\Lambda}\mathrm{H}) = -2.354(50)$ MeV

${}^{4}_{\Lambda}$He and ${}^{4}_{\Lambda}$H results

Λ separation energies for ${}^{4}_{\Lambda}$H

Λ [MeV]	550	600	650	700	Jülich 05	Nijm SC97f	Expt.
$E_{sep}(0^+)$ [MeV]	2.63	2.46	2.36	2.38	1.87	1.60	2.04
$E_{sep}(1^+)$ [MeV]	1.85	1.51	1.23	1.04	2.34	0.54	1.00
ΔE_{sep} [MeV]	0.78	0.95	1.13	1.34	-0.48	0.99	1.05
CSB - 0^+ [MeV]	0.01	0.02	0.02	0.03	-0.01	0.12	0.35
CSB - 1^+ [MeV]	-0.01	-0.01	-0.01	-0.01		-0.01	0.24

Faddeev calculations for ${}^{4}_{\Lambda}$He and ${}^{4}_{\Lambda}$H done by Andreas Nogga, see

- A. Nogga, arXiv:nucl-th/0611081

EFT gives qualitative description of four-body hypernuclei

Strangeness $S = -2$ YY and ΞN interactions

Strangeness $S = -2$ channels in the particle basis:

$$
\begin{aligned}
Q = +2: &\quad \Sigma^+\Sigma^+ \,, \\
Q = +1: &\quad \Xi^0 p, \Sigma^+\Lambda, \Sigma^0\Sigma^+ \,, \\
Q = \ \ 0: &\quad \Lambda\Lambda, \Xi^0 n, \Xi^- p, \Sigma^0\Lambda, \Sigma^0\Sigma^0, \Sigma^-\Sigma^+ \,, \\
Q = -1: &\quad \Xi^- n, \Sigma^-\Lambda, \Sigma^-\Sigma^0 \,, \\
Q = -2: &\quad \Sigma^-\Sigma^- \,.
\end{aligned}
$$

Some experimental knowledge exists for the $Q = 0$ channel:

- $\Delta B_{\Lambda\Lambda} = 1.01 \pm 0.20^{+0.18}_{-0.11}$ MeV (from ${}^{6}_{\Lambda\Lambda}\mathrm{He}$)

  (Takahashi et al. Phys. Rev. Lett. 21 (2001) 212502)
- $\Xi^- p \to \Xi^- p, \Lambda\Lambda$ scattering cross sections at $p_{\mathrm{lab}} = 500$ MeV

  (J.K. Ahn et al., Phys. Lett. B 663 (2006) 214)

LO contact terms for YY and ΞN

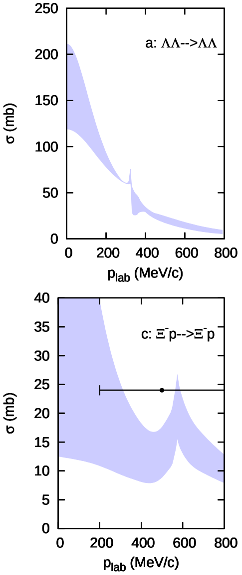
BB contact terms in terms of the $SU(3)_f$ irreducible representations:

$\{8\} \times \{8\} = \{27\} + \{10\} + \{10^*\} + \{8_s\} + \{8_a\} + \{1\}$

S	V	$^1S_0$	$^3S_1$
0	$V^{NN,NN}$	V^{27}	V^{10^*}
-1	$V^{\Lambda N,\Lambda N}$	$\frac{1}{10}\left(9V^{27} + V^{8_s}\right)$	$\frac{1}{2}\left(V^{8_a} + V^{10^*}\right)$
	$V^{\Lambda N,\Sigma N}$	$\frac{3}{10}\left(-V^{27} + V^{8_s}\right)$	$\frac{1}{2}\left(-V^{8_a} + V^{10^*}\right)$
	$V^{\Lambda N,\Lambda N}$	$\frac{1}{10}\left(V^{27} + 9V^{8_s}\right)$	$\frac{1}{2}\left(V^{8_a} + V^{10^*}\right)$
	$V^{\Sigma N,\Sigma N;3/2}$	V^{27}	V^{10}
-2	$V^{\Lambda\Lambda,\Lambda\Lambda}$	$\frac{1}{40}\left(27V^{27} + 8V^{8_s} + 5V^{1}\right)$	–
	$V^{\Xi N,\Xi N;0}$	$\frac{1}{40}\left(12V^{27} + 8V^{8_s} + 20V^{1}\right)$	V^{8_a}
	$V^{\Xi N,\Xi N;1}$	$\frac{1}{5}\left(2V^{27} + 3V^{8_s}\right)$	$\frac{1}{3}\left(V^{10} + V^{10^*} + V^{8_a}\right)$
	etc...		

One additional contact term (V^1) for the $S = -2$ isospin 0 channels

YY and ΞN integrated cross sections



Chiral EFT predictions

Cut off fixed at 600 MeV

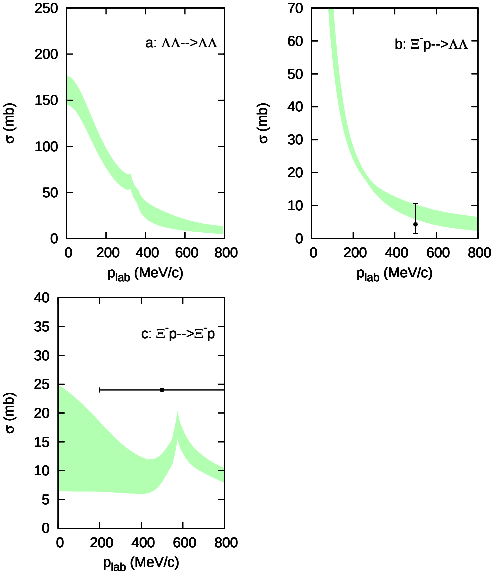
Variation in additional contact term

$$a^{\Lambda\Lambda}_{^1S0} = -1.38, \ldots, -1.83 \text{ fm}$$

$$a^{\Lambda\Lambda}_{^1S0} = -1.32 \text{ fm (ESC04d)}$$

$$a^{\Lambda\Lambda}_{^1S0} = -0.81 \text{ fm (fss2)}$$

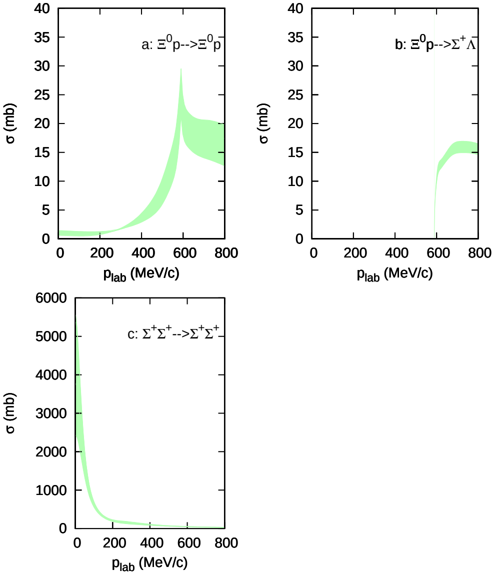
YY and ΞN integrated cross sections



Chiral EFT predictions

Cut off 550 – 700 MeV

YY and ΞN integrated cross sections



Chiral EFT predictions
Cut off 550 – 700 MeV

These channels are independent of the additional contact term

YY and ΞN scattering lengths

	Chiral EFT			
Λ [MeV]	550	600	650	700
$a_s^{\Lambda\Lambda}$	-1.52	-1.52	-1.54	-1.67
$a_t^{\Xi^0 n}$	-0.34	-0.25	-0.20	-0.15
$a_s^{\Xi^0 p}$	0.21	0.19	0.17	0.13
$a_t^{\Xi^0 p}$	0.02	0.00	0.02	0.03
$a_s^{\Sigma^+\Sigma^+}$	-6.23	-7.76	-9.42	-9.27
$C_{1S0}^{\Lambda\Lambda,\Lambda\Lambda}$	-0.0165	0.0000	0.0578	0.0598

Scattering lengths in fm, contact term in $10^4 \mathrm{GeV}^{-2}$.

Summary and outlook

- YN interactions in a chiral EFT approach, based on Weinberg 's power counting, analogous to the NN case
- LO potential (contact terms, one-pseudoscalar-meson exchange) is derived imposing $SU(3)_f$ constraints
- Good description of the empirical YN scattering data was achieved (5 free low-energy coefficients!)
- $SU(3)_f$ extension into the strangeness $S = -2$ channels
  Predictions for YY and ΞN scattering look okay

Work in progress:

- A combined NN and YN study in a chiral EFT, starting with a next-to-leading order (NLO) calculation
  (more and precise YN data would be desirable)