Evaluation of the Asymmetry in the Weak Nonmesonic Hypernuclear Decay

Cesar Barbero

Departamento de Física, Facultad de Ciencias Exactas

Universidad Nacional de La Plata, La Plata, Argentina

Alfredo P. Galeão

Instituto de Física Teórica, Universidade Estadual Paulista, ão Paulo, Brazil

Francisco Krmpotić

Instituto de Física, Universidade de São Paulo, São Paulo, Brazil

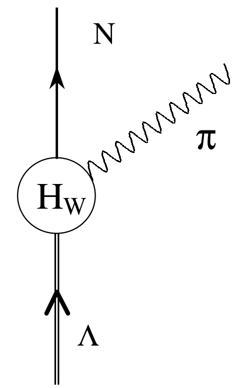
Instituto de Física La Plata, CONICET, La Plata, Argentina

Outline

- Introduction
  - Λ-Hyperon Weak Decay
  - Hypernuclear Mesonic Weak Decay (MWD)
  - Hypernuclear Nonmesonic Weak Decay (NMWD)
  - Hypernuclear Asymmetry and Parameter a_Λ
- Motivations for NMWD and a_Λ
- a_Λ from Spin Observables in reaction: $p + n \rightarrow p + \overrightarrow{\Lambda}$ (Nabetani-Ogaito-Sato-Kishimoto (NOSK) Formula)
- a_Λ from Shell Model (SM) for NMWD: ${}^{12}_{\Lambda}\overrightarrow{C} \rightarrow {}^{11}B + p + n$
- s-Wave Approximation in SM $\Longleftrightarrow$ NOSK Formula

Λ-Hyperon Weak Decay

$\cong 100$ % of the time Λ decays by the $\Lambda \to N\pi$ weak-mesonic mode



$$\Lambda \to \begin{cases} p + \pi^- & (64.1\%) \\ n + \pi^0 & (35.7\%), \end{cases}$$

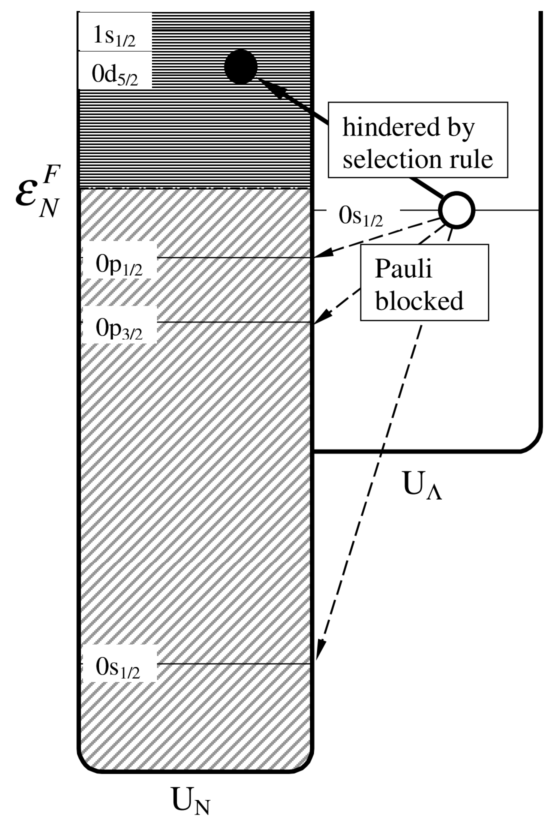
Energy released:

$$Q_0 = M_\Lambda - M_N - m_\pi \cong 37 \; MeV$$

Hypernuclear-Mesonic-Weak-Decay (MWD)

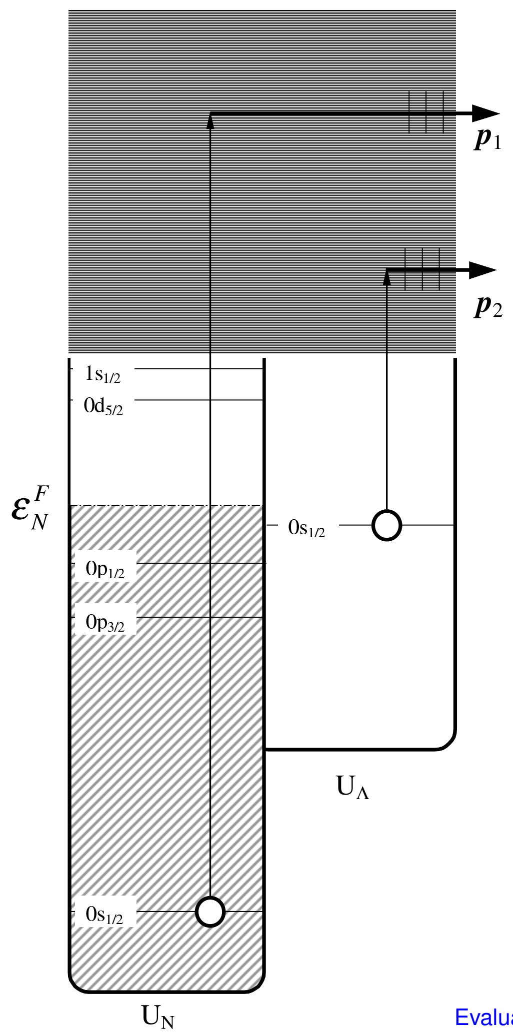
Is blocked by the Pauli Principle, and the Q-value is very small:

$$Q_M = M_\Lambda - M_N - m_\pi + \varepsilon_\Lambda - \varepsilon_N^\uparrow < Q_0$$



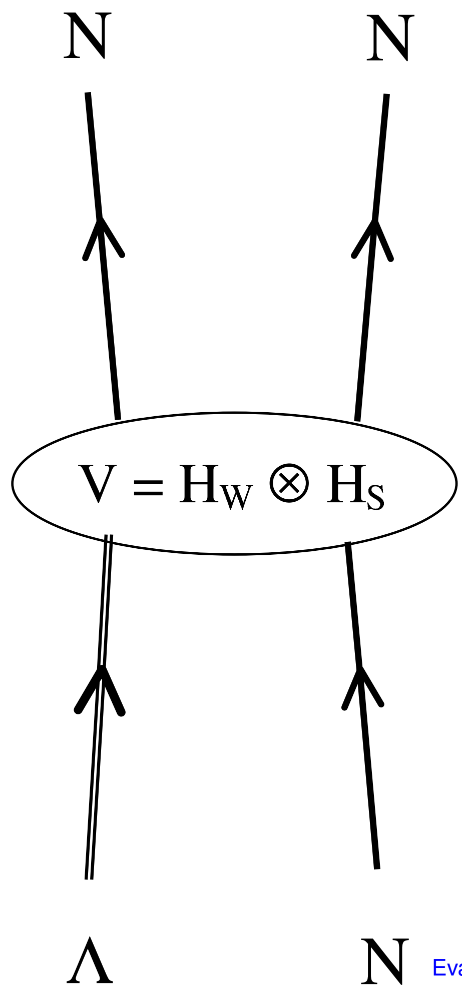
Hypernuclear-Nonmesonic-Weak-Decay (NMWD)

New NMWD channel $\Lambda N \to NN$ become open inside the nucleus:
It is not Pauli blocked, and the Q-value is large:
$Q_{NM} = M_\Lambda - M_N + \varepsilon_\Lambda + \varepsilon_N^{\downarrow} \sim 120 - 135$ MeV



Why Hypernuclear-Nonmesonic-Weak-Decay?

The NMWD offers a unique opportunity to gain insight into the fundamental aspects of the two-fermion strangeness changing weak interaction: In the decay $\Lambda + N \to N + N$ one meson is exchange between a weak vertex $\mathcal{H}_W$ and a strong vertex $\mathcal{H}_S$.



Hypernuclear Asymmetry

Λ-hypernuclei produced in a (π^+, K^+) reaction, end up with considerable vector polarization along the direction normal to the reaction plane.
The initial mixed state, with spin J_I, from which the hypernucleus decays is described by the Density Matrix

$$\rho(J_I) = \frac{1}{2J_I+1}\left[1 + \frac{3}{J_I+1}\mathbf{P}_V \cdot \mathbf{J}_I\right] ,$$

Vector Polarization:

$$\mathbf{P}_V = P_V \cdot \hat{\mathbf{n}} \qquad \hat{n} = \frac{(\ p_{\pi^+} \times \ p_{K^+})}{|\ p_{\pi^+} \times \ p_{K^+}|} \perp \text{ to the reaction plane}$$

Intrinsic Λ asymmetry parameter: a_Λ

Angular Distribution of protons with momentum $\mathbf{p}_p$ from the decay of the polarized mixed state has the form (our starting point):

$$\frac{d\Gamma[\rho(J_I) \to \hat{\mathbf{p}}_p]}{d\Omega_p} = \frac{\Gamma_p}{4\pi} \left(1 + A_V \, \mathbf{P}_V \cdot \hat{\mathbf{p}}_p\right) \, ,$$

Γ_p: Full proton-induced Decay Rate
A_V: Vector Hypernuclear Asymmetry

$$a_\Lambda = \begin{cases} A_V & \text{for} \quad J_I = J_C + 1/2 \, , \\ -\frac{J_I+1}{J_I} A_V & \text{for} \quad J_I = J_C - 1/2 \, , \end{cases}$$

a_Λ: Intrinsic Λ asymmetry parameter: does not depend on the hypernuclear spin $\vec{J}_I = \vec{J}_C + \vec{j}_\Lambda$

Why Asymmetry Parameter a_Λ?

- Measurements favour $a_\Lambda({}^5_\Lambda\vec{He}) > 0$ and $a_\Lambda({}^{12}_\Lambda\vec{C}) < 0$
- Calculations within the one-meson-exchange model (OMEM) yield almost the *same* negative value for both hypernuclei.

Ref. and Model	${}^5_\Lambda\vec{\mathrm{He}}$	${}^{12}_\Lambda\vec{\mathrm{C}}$
Sasaki et al. 1999: $\pi + K + \mathrm{Direct\ Quark}$	-0.68	
Parreño et al. 2002: $\pi + \rho + K + K^* + \omega + \eta$	-0.68	-0.73
Itonaga et al. 2003: $\pi + K + \omega + 2\pi/\rho + 2\pi/\sigma$	-0.33	
Barbero et al. 2005: $\pi + \rho + K + K^* + \omega + \eta$	-0.54	
KEK–E160 1992		-0.9 ± 0.3
KEK–E278 2000	0.24 ± 0.22	
KEK–E508 (prel.) 2004		-0.44 ± 0.32
KEK–E462 (prel.) 2004	0.07 ± 0.08	

Nabetani-Ogaito-Sato-Kishimoto Formula

Spin observables in the $pn \to p\Lambda$ reaction: PRC60 (1999) 017001

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\Omega}|_{unpol.}[1 + a_\Lambda \mathbf{P}_\Lambda \cdot \hat{\mathbf{p}}\,]$$

$\mathbf{P}_\Lambda$: polarization vector of Λ
$\hat{\mathbf{p}}$: momentum of the initial proton

$$a_\Lambda = 2\sqrt{3}\,\frac{\Re[ae^* - b\,(c - \sqrt{2}\,d)^*/\sqrt{3}\, + f\,(\sqrt{2}\,c + d)^*]}{|a|^2 + |b|^2 + 3\,(|c|^2 + |d|^2 + |e|^2 + |f|^2)}\,,$$

$$a = \langle {}^1\mathrm{S}_0|V|{}^1\mathrm{S}_0\rangle \qquad b = \langle {}^3\mathrm{P}_0|V|{}^1\mathrm{S}_0\rangle \qquad c = \langle {}^3\mathrm{S}_1|V|{}^3\mathrm{S}_1\rangle,$$

$$d = \langle {}^3\mathrm{D}_1|V|{}^3\mathrm{S}_1\rangle \qquad e = \langle {}^1\mathrm{P}_1|V|{}^3\mathrm{S}_1\rangle \qquad f = \langle {}^3\mathrm{P}_1|V|{}^3\mathrm{S}_1\rangle.$$

s-Wave Approximation (sWA)

- NOSK formula was derived within the sWA: only the s-wave production for the $p\Lambda$ final states was considered:

$\longrightarrow$ NOSK formula can be used for the NMWD in:

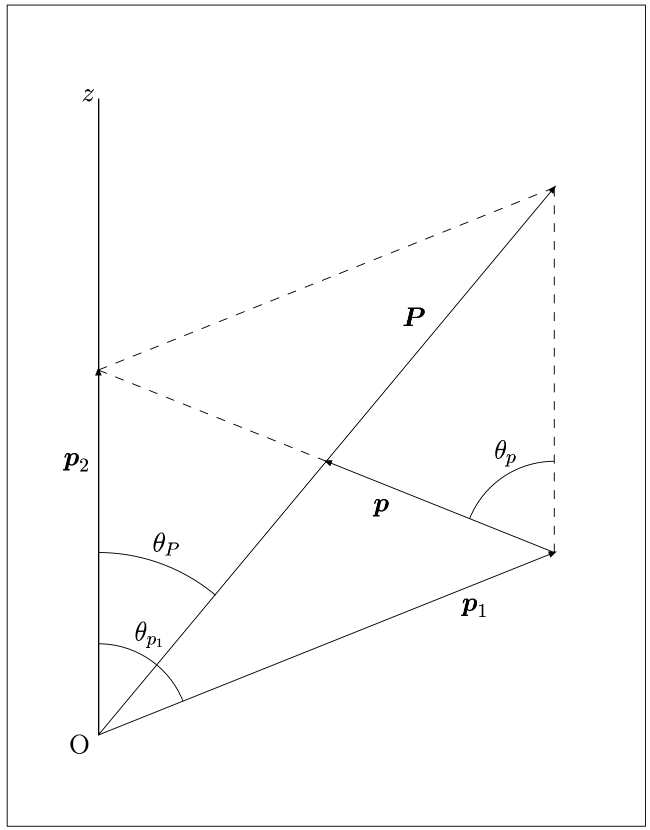
- Fermi gas model: initial Λ-hyperon always in a relative s-state with respect to any of the nucleons within the hypernucleus
- $1s_{1/2}$-shell hypernuclei : the initial $p\Lambda$ system is always in the relative s-wave state ($^{5}_{\Lambda}$He)

$\longrightarrow$ NOSK formula can not be used ???? for the NMWD in:

- Shell Model description: the hyperon stays in the $1s_{1/2}$ orbital, and proton can occupy the orbitals $1s_{1/2}, 1p_{3/2}, 1p_{1/2}, \cdots$: the initial $p\Lambda$ system is not always in the relative s-wave state ($^{12}_{\Lambda}$C)

Shell Model Evaluation of a_Λ for ${}^{5}_{\Lambda}$He and ${}^{12}_{\Lambda}$C

Coordinate system for the outgoing particles: $1 \rightarrow n, 2 \rightarrow p$



Representation to relative and total momenta:

$$\mathbf{p} = (\mathbf{p}_2 - \mathbf{p}_1)/2\,, \qquad \mathbf{P} = \mathbf{p}_2 + \mathbf{p}_1$$

a_Λ does not depend on initial and final spins:

$$a_\Lambda = \frac{\omega_1}{\omega_0} \,,$$

$$\begin{aligned}
\omega_\kappa \quad &= \quad (-)^\kappa \frac{8}{\sqrt{2\pi}} \hat{\kappa}^{-1} \sum_{j_p} \int dU_{j_p} \, Y_{\kappa 0}(\theta_p, 0) \\
&\times \quad \sum_{TT'} (-)^{T+T'} \sum_{LS} \sum_{l\lambda J} \sum_{l'\lambda' J'} i^{l-l'} \, (-)^{\lambda+\lambda'+S+L+j_p+\frac{1}{2}} \\
&\times \quad \hat{l}\hat{l'}\hat{\lambda}\hat{\lambda'}\hat{J}^2\hat{J'}^2 \, (l0l'0|\kappa 0) \\
&\times \quad \left\{ \begin{array}{ccc} \kappa & 1/2 & 1/2 \\ j_p & J & J' \end{array} \right\} \left\{ \begin{array}{ccc} \kappa & J' & J \\ S & \lambda & \lambda' \end{array} \right\} \left\{ \begin{array}{ccc} l' & l & \kappa \\ \lambda & \lambda' & L \end{array} \right\} \\
&\times \quad \mathcal{M}(plPL\lambda SJT; j_\Lambda j_p) \mathcal{M}^*(pl'PL\lambda' SJ'T'; j_\Lambda j_p),
\end{aligned}$$

$$\int dU_{j_p} \cdots = \int d\cos\theta_{p_1} \int p_2^2\, dp_2 \int p_1^2\, dp_1$$

$$\times \quad \delta\left(\frac{p_1^2}{2M_N} + \frac{p_2^2}{2M_N} + \frac{|\mathbf{p}_1 + \mathbf{p}_2|^2}{2M_F} - \Delta_{j_p}\right) \cdots; \; \Delta_{j_p} = M_\Lambda - M_N + \epsilon_{j_\Lambda} + \epsilon_{j_p}$$

$$\mathcal{M}(plPL\lambda SJT; \mathrm{j}_\Lambda j_p) = \frac{1}{\sqrt{2}} \left[1 - (-)^{l+S+T}\right] (plPL\lambda SJT|V|\mathrm{j}_\Lambda j_p J)$$

$$|\mathrm{j}_\Lambda j_p J) \quad + \quad \left\{ \begin{array}{cc} jj - LS & \text{recoupling} \\ \multicolumn{2}{c}{\text{Moshinsky Transformation}} \end{array} \right\} \rightarrow |\mathrm{nlL}\lambda SJ)$$

$j_p = s_{1/2} \rightarrow \mathrm{l} = 0, \mathrm{L} = 0$: For ${}^{5}_{\Lambda}$He and ${}^{12}_{\Lambda}$C

$j_p = p_{3/2} \rightarrow \mathrm{l} = 0$, $\mathrm{L} = 1$, and $\mathrm{l} = 1$, $\mathrm{L} = 0$: ${}^{12}_{\Lambda}$C

sWA within the Shell Model for a_Λ in ${}^{12}_{\Lambda}$C

- $\Delta_{j_p} = M_\Lambda - M_N + \epsilon_{j_\Lambda} + \epsilon_{j_p} \rightarrow \Delta = M_\Lambda - M_N$

$$\omega_\kappa = \frac{8}{\sqrt{\pi}} \sum_{j_p \mathsf{l} \mathsf{L}} \int dU Y_{\kappa 0}(\theta_p, 0) \mathcal{O}(P; \mathsf{L}) \mathcal{I}_\kappa(p; j_p, \mathsf{l})$$

$$\mathcal{O}(P; \mathsf{L}) \equiv (P\mathsf{L}|1\mathsf{L})^2;\ (PL|\mathsf{NL}) = \delta_{L,\mathsf{L}} \int R^2 dR j_L(PR) \mathcal{R}_{\mathsf{NL}}(R),$$

- $\mathcal{I}_\kappa(p; j_p = p_{3/2}, 0) = \mathcal{I}_\kappa(p; j_p = s_{1/2}, 0) \equiv \mathcal{I}_\kappa(p; 0)$
- $\mathcal{I}_0(p; j_p = p_{3/2}, 1) \cong 0$
- $\mathcal{O}(P; \mathsf{L} = 0) \cong \mathcal{O}(P; \mathsf{L} = 1)$

$$a_\Lambda = \frac{\omega_1(j_p = p_{3/2}) + \omega_1(j_p = 1s_{1/2})}{\omega_0(j_p = p_{3/2}) + \omega_0(j_p = s_{1/2})} = \frac{\omega_1(j_p = s_{1/2})}{\omega_0(j_p = s_{1/2})}$$

NOSK-like Formula for ${}^{5}_{\Lambda}$He and ${}^{12}_{\Lambda}$C

$$\omega_\kappa = \frac{16}{\sqrt{\pi}} \int dU Y_{\kappa 0}(\theta_p, 0) \mathcal{I}_\kappa(p; 0) \mathcal{O}(P; 0)$$

$$\begin{aligned} \mathcal{I}_0(p;0) &= |\mathsf{a}|^2 + |\mathsf{b}|^2 + 3(|\mathsf{c}|^2 + |\mathsf{d}|^2 + |\mathsf{e}|^2 + |\mathsf{f}|^2) \\ \mathcal{I}_1(p;0) &= 2\Re[\mathsf{a}\mathsf{e}^* - \mathsf{b}(\mathsf{c} - \sqrt{2}\mathsf{d})^*/\sqrt{3} + \mathsf{f}(\sqrt{2}\mathsf{c} + \mathsf{d})^*] \end{aligned}$$

$\mathsf{a} = \langle 000|V|000\rangle_{SM}, \cdots \mathsf{f} = \langle 011|V|101\rangle_{SM}$, as in NOSK-formula:

$$\mathcal{M}(plP0, lSJT; j_\Lambda, j_p) = (-)^{T+1}(P0|10)\langle lSJ|V|0JJ\rangle_{SM}.$$

NOSK Formula for ${}^{5}_{\Lambda}$He and ${}^{12}_{\Lambda}$C

$$a_\Lambda = \frac{\omega_1}{\omega_0};\ \omega_\kappa = \frac{16}{\sqrt{\pi}} \int dU Y_{\kappa 0}(\theta_p, 0) \mathcal{I}_\kappa(p; 0) \mathcal{O}(P; 0)$$

- decay is basically back to back: $Y_{1,0}(\theta_p, 0) \cong Y_{1,0}(0, 0) = \sqrt{3/4\pi}$
- $\mathcal{I}_\kappa(p; 0) \cong \mathcal{I}_\kappa(p = p_\Delta; 0); p_\Delta = \sqrt{M_N \Delta} \cong 400$ MeV

$$a_\Lambda = \sqrt{3} \frac{\mathcal{I}_1(p = p_\Delta; 0)}{\mathcal{I}_0(p = p_\Delta; 0)},$$

OMEM ($\pi, \eta, K, \rho, \omega, K^*$) mesons

$$\begin{aligned}
\text{a} &= \frac{1}{\sqrt{2}}\left[C_1^0 + C_0^0 - 3(S_1^0 + S_0^0)\right] \\
\text{c} &= \frac{1}{\sqrt{2}}\left[S_0^0 + C_0^0 - 3(S_1^0 + C_1^0)\right] \\
\text{d} &= 2(3T_1^{20} - T_0^{20}) \\
\text{b} &= -\frac{1}{\sqrt{2}}(P_\pi^{10} + P_{K_1}^{10} + P_\eta^{10} + P_{K_0}^{10}) + \sqrt{2}(\tilde{P}_{K_1^*}^{10} + \tilde{P}_{K_0^*}^{10}) \\
\text{e} &= -\frac{1}{\sqrt{6}}\left[3(P_\pi^{10} + P_{K_1}^{10} + 2\tilde{P}_{K_1^*}^{10}) - P_\eta^{10} - P_{K_0}^{10} - 2\tilde{P}_{K_0^*}^{10}\right] \\
\text{f} &= -\frac{1}{\sqrt{3}}\left[P_\pi^{10} - P_{K_1}^{10} + P_\eta^{10} - P_{K_0}^{10}\right]
\end{aligned}$$

$S, C, T, P, \tilde{P}$ defined in PRC66, 055209 (2002)

Exact (a_Λ^{SM}) and approximate (a_Λ^{NOSK}) results

Model	${}^5_\Lambda$He		${}^{12}_\Lambda$C	
	a_Λ^{SM}	a_Λ^{NOSK}	a_Λ^{SM}	a_Λ^{NOSK}
π	-0.4354	-0.4351	-0.4324	-0.4501
(π, η, K)	-0.5652	-0.5852	-0.5526	-0.5860
$\pi + \rho$	-0.2449	-0.2665	-0.2379	-0.2554
$(\pi, \eta, K, \rho, \omega, K^*)$	-0.5117	-0.5131	-0.5088	-0.5306

Conclusion: The NOSK formula is a good approximation for the asymmetry parameter a_Λ both in ${}^5_\Lambda$He and ${}^{12}_\Lambda$C

More on a_Λ^π

$$a_\Lambda^\pi = -\frac{2\left[\left(2\mathsf{T}_\pi^{20} - \mathsf{S}_\pi^{0}\right)\mathsf{P}_\pi^{10}\right]}{18(\mathsf{T}_\pi^{20})^2 + (\mathsf{P}_\pi^{10})^2 + 3(\mathsf{S}_\pi^{0})^2} \cong -\frac{4\mathsf{T}_\pi^{20}\mathsf{P}_\pi^{10}}{18(\mathsf{T}_\pi^{20})^2 + (\mathsf{P}_\pi^{10})^2} \cong -0.44$$

Matrix element for π (MeV$^{-1/2}$)	${}^{5}_{\Lambda}He$	${}^{12}_{\Lambda}C$
T_π^{20}	-3.2402	-3.7132
P_π^{10}	-8.0573	-10.0379
S_π^{0}	0.3876	0.4088

Conclusion: a_Λ^π is large and negative in the OPEM due to the interplay between the PC tensor (T_π^{20}) and PV dipole (P_π^{10}) matrix elements. The inclusion of other mesons does not modify the above picture to a great extent.