In-medium properties of pion
and
partial restoration of chiral symmetry
in nuclear medium

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Question:

Does really partial restoration of chiral symmetry take place ?

How can we conclude it from experimental observations ?

How can we implement it to theoretical calculations ?

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Introduction

Chiral symmetry

- spontaneously broken by vacuum
- believed to be restored in high densities and/or temperatures

Nuclear matter

- density is not enough for the restoration
- but finite density system

We expect that

- **Broken χS is partially restored in nuclear medium**
  (effective reduction of the chiral condensate)
- **PRχS observed in medium modif. of hadron properties**

medium modifications of hadron properties

- consequence of complex dynamics in hadron-nucleus system

some universal nature of medium modifications comes from symmetry

important to

separate symmetry consequence out of the complex dynamics

Introduction

Exp. studies of **pionic atom** and π**-nucleus elastic scattering** suggest reduction of decay constant in nucleus

$$F_t^{*2}/F^2 < 1$$

K. Suzuki et al., PRL92 (04) 072302.
E. Friedman et al., PRL93 (04) 122302.
W. Weise, NPA690 (01) 98.

Question

how to conclude **reduction of quark condensate** in medium

$$\langle \bar{q}q \rangle^* / \langle \bar{q}q \rangle < 1$$

In-medium GOR relation

$$F_t^{*2} m_\pi^{*2} = -2 m_q \langle \bar{q}q \rangle^*$$

$$\frac{F_t^{*2}}{F^2} \frac{m_\pi^{*2}}{m_\pi^2} = \frac{\langle \bar{q}q \rangle^*}{\langle \bar{q}q \rangle}$$

Using low energy theorem **model-independently**,
we show **another relation** in chiral limit

$$\frac{F_t^*}{F} \frac{Z^{*1/2}}{Z^{1/2}} = \frac{\langle \bar{q}q \rangle^*}{\langle \bar{q}q \rangle}$$

in-medium quark condensate is expressed as normalization of pion field together with in-medium pion decay constant.

Derivation of the scaling law

Jido, Hatsuda, Kunihiro, in preparation.

$$\frac{F_t^*}{F}\frac{Z^{*1/2}}{Z^{1/2}} = \frac{\langle \bar{q}q\rangle^*}{\langle \bar{q}q\rangle}$$

- two flavor χS
- chiral limit
- operator relation
  (independent of states)

Chiral four vector $SU(2)_L \otimes SU(2)_R$

pseudoscalar $\phi_5^a = \bar{q}\gamma_5 t^a q$

scalar $\phi = \bar{q}t^0 q$

axial trans.

$$[Q_5^a, \phi_5^b] = -i\delta^{ab}\phi$$

PCAC relation

$$\partial^\mu A_\mu^a(x) = 0 \quad \text{(chiral limit)}$$

- in-medium correlation function

$$\Pi^{ab}(q) = \int d^4x e^{iq\cdot x}\partial^\mu \langle\Omega|T[A_\mu^a(x)\phi_5^b(0)]|\Omega\rangle \quad \text{symmetric matter}$$

used for proof of emergence of Nambu-Goldstone boson in vacuum

Derivation of the scaling law

Jido, Hatsuda, Kunihiro, in preparation.

$$\Pi^{ab}(q) = \int d^4x e^{iq\cdot x}\partial^\mu \langle \Omega | T[A_\mu^a(x)\phi_5^b(0)]|\Omega\rangle$$ symmetric matter

Ward-Takahashi identity

$$\partial^\mu T[A_\mu^a(x)\phi_5^b(0)] = T[\partial^\mu A_\mu^a(x)\phi_5^b(0)] + \delta(x_0)[A_0^a(x), \phi_5^b(0)]$$

vanish in chiral limit $[Q_5^a, \phi_5^b] = -i\delta^{ab}\phi$

right hand side

$$\lim_{q_\mu \to 0} \Pi^{ab}(q_\mu) = \int d^4x\, \delta(x_0)\langle \Omega | [A_0^a(x), \phi_5^b(0)]|\Omega\rangle = -i\delta^{ab}\langle \bar{q}q\rangle^*$$

The correlation function is written as the in-medium quark condensate in the soft limit.

Jido, Hatsuda, Kunihiro, in preparation.

Derivation of the scaling law

$$\Pi^{ab}(q) = \int d^4x e^{iq\cdot x} \partial^\mu \langle \Omega | T[A^a_\mu(x) \phi^b_5(0)] | \Omega \rangle$$ symmetric matter

In soft limit, correlation function saturates zero mode propagation.

zero modes in nuclear medium

- ***pionic mode*** as a consequence of spontaneous χSB
- ***particle-hole excitations*** without pion production

Admixture states are physically observed.

Let us consider such a zero mode mixed state
as the intermediate state in the correlator.

$|\Omega_5\rangle$ in-medium zero mode with pionic quantum number ($I=1$, $J^P=0^-$)

Derivation of the scaling law

Jido, Hatsuda, Kunihiro, in preparation.

$$\Pi^{ab}(q) = \int d^4x e^{iq\cdot x}\partial^\mu \langle\Omega|T[A_\mu^a(x)\phi_5^b(0)]|\Omega\rangle$$ symmetric matter

Matrix elements of $\phi_5^a(x)$ and $A_\mu^a(x)$

Lorentz invariance

n_μ characterizing nuclear matter

$$\langle\Omega|\phi_5^a(x)|\Omega_5^b(q)\rangle = \delta^{ab}Z^{*1/2}e^{-iq\cdot x}$$

$$\langle\Omega|A_\mu^a(x)|\Omega_5^b(q)\rangle = \delta^{ab}i\left(-\frac{q^2}{(q\cdot n)}n_\mu + q_\mu\right)F^* e^{-iq\cdot x}$$

Z^*, F^*, N
functions of $q^2, (n\cdot q)$

Taking the frame $n_\mu = (1,0,0,0)$

pion dispersion rel. $q_0^2 - v_\pi^2\vec{q}^{\,2} = 0$

temporal

$$\langle\Omega|A_0^a(x)|\Omega_5^b(q)\rangle = \delta^{ab}iq_0\frac{1}{v_\pi^2}F^* e^{-iq\cdot x} \longrightarrow F_t^*$$

spatial

$$\langle\Omega|A_i^a(x)|\Omega_5^b(q)\rangle = \delta^{ab}iq_i F^* e^{-iq\cdot x} \longrightarrow F_s^*$$

$$\boxed{F_s^*/F_t^* = v_\pi^2}$$

linear dependence of energy-momentum

Derivation of the scaling law

Jido, Hatsuda, Kunihiro, in preparation.

$$\Pi^{ab}(q) = \int d^4x e^{iq\cdot x} \partial^\mu \langle \Omega | T[A^a_\mu(x)\phi^b_5(0)] | \Omega \rangle$$ symmetric matter

Pion contribution to correlation function in soft limit

$$\lim_{q_\mu \to 0} \Pi^{ab}(q) = \lim_{q_\mu \to 0} \left(i\delta^{ab} \frac{q_0^2 F_t^* - \vec{q}^{\,2} F_s^*}{q_0^2 - v_\pi^2 \vec{q}^{\,2}} Z^{*1/2} + \cdots \right) = \delta^{ab} i F_t^* Z^{*1/2}$$

Combing this with the previous result, we obtain

$$F_t^* Z^{*1/2} = -\langle \bar{q} q \rangle^*$$

Taking ratio to counterpart in vacuum, finally we get

$$\frac{F_t^*}{F} \frac{Z^{*1/2}}{Z^{1/2}} = \frac{\langle \bar{q} q \rangle^*}{\langle \bar{q} q \rangle}$$

exact in chiral limit

Pion Wavefunction Renormalization

$$D^*_\pi(q) = \frac{1}{3}\int d^4x\, e^{iq\cdot x}\langle\Omega|T[\phi^a_5(x)\phi^a_5(0)]|\Omega\rangle$$ symmetric matter

pion pole at $q_0^2 - v_\pi^2\vec{q}^{\,2} = 0$ with residue Z^*

Equivalently collecting all the in-medium corrections to self-energy

$$D^*_\pi(q) = \frac{iZ}{q^2 - \Sigma_\pi(q_0,\vec{q})} + \cdots$$ with in-vacuum Z

Wavefunction renormalization

$$Z^* = Z\left(1 - \left.\frac{\partial\Sigma_\pi(q_0,\vec{q})}{\partial q_0^2}\right|_{q_\mu=0}\right)^{-1}$$

Pion Wavefunction Renormalization

$$D^*_\pi(q) = \frac{1}{3}\int d^4x\, e^{iq\cdot x}\langle\Omega|T[\phi^a_5(x)\phi^a_5(0)]|\Omega\rangle$$ symmetric matter

Wavefunction renormalization

$$Z^* = Z\left(1 - \left.\frac{\partial\Sigma_\pi(q_0,\vec{q})}{\partial q_0^2}\right|_{q_\mu=0}\right)^{-1}$$

can be evaluated by πN scattering in linear density approximation

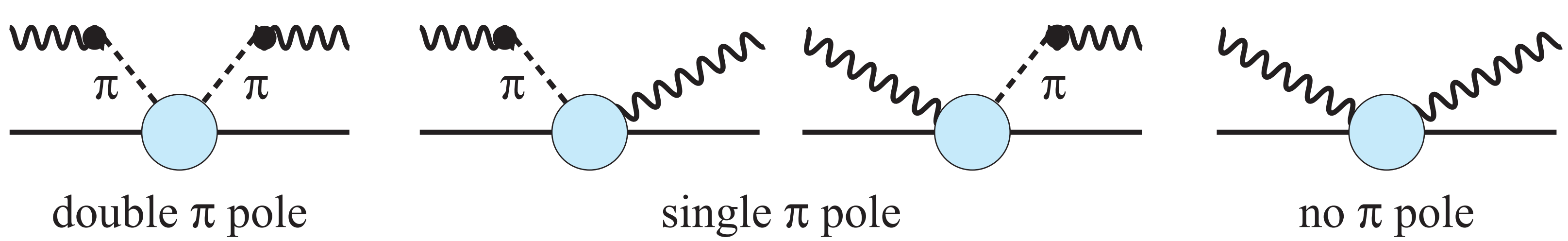
$$\Sigma_\pi(q_0) = -\rho T^{(+)}_{\pi N}(q_0)$$

πN scattering amplitude is given by reduction formula

$$T^{(+)}_{\pi N}(\nu,\tilde{\nu};k^2,k'^2) = \frac{i}{3}Z^{-1}k^2k'^2\int d^4x\, e^{ik\cdot x}\langle N|T\phi^a_5(x)\phi^a_5(0)|N\rangle$$

including off-shell extrapolation

Off-shell extrapolation is unique, once interpolating field is fixed, and consistent with low energy theorems obtained from commutation relation involving the pseudoscalar density.



Pion Wavefunction Renormalization

Wavefunction renormalization

$$Z^* = Z\left(1 - \frac{\partial \Sigma_\pi(q_0, \vec{q})}{\partial q_0^2}\bigg|_{q_\mu=0}\right)^{-1}$$

linear density approximation

$$\Sigma_\pi(q_0) = -\rho T_{\pi N}^{(+)}(q_0)$$

chiral expansion of amplitude

$$T_{\pi N}^{(+)}(q_0) = \alpha + \beta q_0^2$$

α: explicit χSB

β: chiral limit

wavefunction renormalization in linear density

$$Z^*/Z = 1 - \beta\rho$$

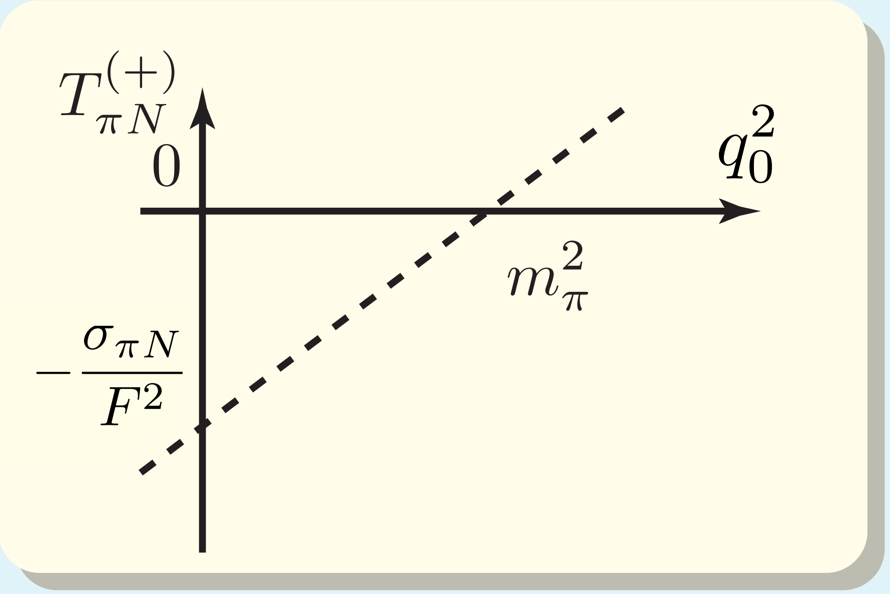
the sign of β can be extracted by knowledge of πN scattering

Weinberg point (soft limit) $\quad T_{\pi N}^{(+)}(0) = -\frac{\sigma_{\pi N}}{F^2}$

Threshold $\quad T_{\pi N}^{(+)}(m_\pi) \simeq 0$

scattering length $\quad a_{\pi N} = (0.0016 \pm 0.0013) m_\pi^{-1}$

positive slope $\quad \beta > 0 \qquad Z^*/Z < 1$



Pion Wavefunction Renormalization

Wavefunction renormalization

linear density approximation

chiral

This conclusion does not change as long as scattering length is

larger than -0.050 m_π^{-1} for $\sigma_{\pi N} \simeq 45$ MeV

-0.067 m_π^{-1} for $\sigma_{\pi N} \simeq 60$ MeV

scaling law

$$\frac{F_t^*}{F}\frac{Z^{*1/2}}{Z^{1/2}} = \frac{\langle \bar{q}q \rangle^*}{\langle \bar{q}q \rangle}$$

pionic atom
π-nucleus scattering

$$F_t^{*2}/F^2 < 1$$

πN scattering

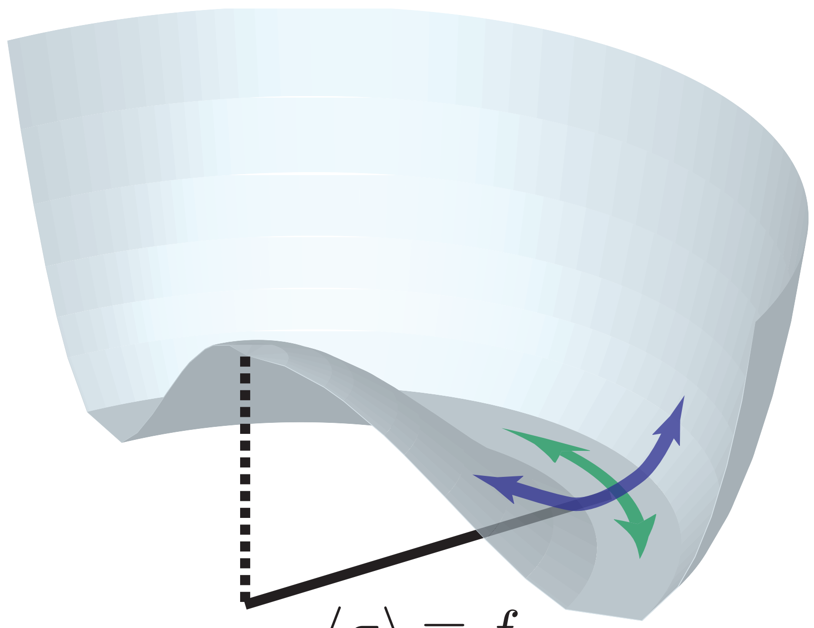
$$Z^{*1/2}/Z^{1/2} < 1$$

$$\Rightarrow \quad \langle \bar{q}q \rangle^* / \langle \bar{q}q \rangle < 1$$

positive slope $\beta > 0$ $\quad Z^*/Z < 1$

Nonlinear sigma model

Importance of field renormalization



Success of low energy effective theroy:

based on decomposition of degrees of freedom,

in consistent way with original symmetry,

into two directions:

angular direction: pi (Nambu-Goldstone) mode

radial direction: sigma mode

Angular variable: dimensionless
while meson field has energy dimension.

$$U = \exp\left[\frac{i\vec{\pi}\cdot\vec{\tau}}{f}\right]$$

The scale, the normalization of the pion field, is determined by the strength of the spontaneous breaking, the chiral condensate.

Therefore, when the partial restoration takes place with shift of chiral condensate, we need to renormalize the pion field. Consequently the decay constant changes within the nonlinear sigma model.

Jido, Hatsuda, Kunihiro, PRD63, 011901(R)

Conclusion

qualitative argument of quark condensate in nuclear medium using low energy theorem

propose new scaling law in chiral limit $\dfrac{F_t^*}{F}\dfrac{Z^{*1/2}}{Z^{1/2}} = \dfrac{\langle\bar{q}q\rangle^*}{\langle\bar{q}q\rangle}$

pionic atom
π-nucleus scattering

$$F_t^{*2}/F^2 < 1$$

πN scattering

$$Z^{*1/2}/Z^{1/2} < 1$$

$$\Rightarrow \quad \langle\bar{q}q\rangle^*/\langle\bar{q}q\rangle < 1$$

for qualitative arguments,
need further studies beyond chiral limit