Nucleon form factors and structure functions in lattice QCD with dynamical DWF quarks

Shigemi Ohta *†‡ [RBC and UKQCD DWF intercollaboration]

June 5, 2007, INPC 2007, Tokyo Japan

RIKEN-BNL-Columbia (RBC) and UKQCD collaborations have produced some dynamical DWF ensembles:

- RBC 2-flavor: DBW2 and DWF, $a^{-1} = 1.7$ GeV, $m_{\rm res} = 0.00137(5)$, 2 fm across:
  - $m_{\rm sea} = 0.04$, 0.03, and 0.02; $m_\pi = 0.7$, 0.6 and 0.5 GeV.
- RBC/UKQCD (2+1)-flavor, Iwasaki and DWF, $a^{-1} = 1.6$ GeV, $m_{\rm res} = 0.00308(3)$, 3 fm across:
  - $m_{\rm strange} = 0.04$ and $m_{\rm up,down} = 0.03$, 0.02, 0.01, 0.005; $m_\pi = 0.6$, 0.5, 0.4 and 0.3 GeV.

Successfully describing hadron physics from its first principle with flavor and chiral symmetries: f_π, f_K, B_K, ϵ'/ϵ, m_N, $m_{N'}$, ...

Here we report some nucleon isovector form factors and moments of structure functions,

- with domain wall fermions (Darn-good, Wonderful, Fantastic[1]) which preserves almost exact chiral and flavor symmetries,
- and makes the non-perturbative renormalization of the relevant currents simple and accurate.

Based on the works by HueyWen Lin, Takeshi Yamazaki, Kostas Orginos, Shoichi Sasaki, and Tom Blum.

---

*Inst. Particle and Nuclear Studies, KEK, Tsukuba, Ibaraki 305-0801, Japan

†Physic Department, Sokendai Graduate University of Advanced Studies, Tsukuba, Ibaraki 305-0801, Japan

‡RIKEN BNL Research Center, Upton, NY 11973, USA

[1] Stephen Sharpe, hep-lat/0706.0218.

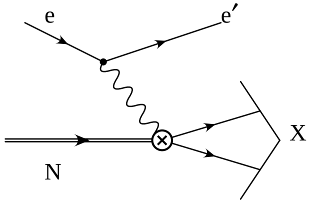
The easiest are the vector and axial charges, known from neutron β decay, $g_V = G_F \cos\theta_c$ and $g_A/g_V = 1.2695(29)$[2]:

- $g_V \propto \lim_{q^2\to 0} g_V(q^2)$ with $\langle n|V_\mu^-(x)|p\rangle = i\bar{u}_n[\gamma_\mu g_V(q^2) + q_\lambda \sigma_{\lambda\mu} g_T(q^2)]u_p e^{-iqx}$,
- $g_A \propto \lim_{q^2\to 0} g_A(q^2)$ with $\langle n|A_\mu^-(x)|p\rangle = i\bar{u}_n\gamma_5[\gamma_\mu g_A(q^2) + q_\mu g_P(q^2)]u_p e^{-iqx}$.

Their momentum-transfer dependence and the induced tensor and pseudoscalar form factors are calculable as well.

Structure functions: measured in deep inelastic scatterings (and RHIC/Spin):

- DIS



$$\begin{aligned} \left|\frac{\mathcal{A}}{4\pi}\right|^2 &= \frac{\alpha^2}{Q^4} l^{\mu\nu} W_{\mu\nu} \\ W^{\mu\nu} &= W^{[\mu\nu]} + W^{\{\mu\nu\}} \end{aligned}$$

$$\begin{aligned} W^{\{\mu\nu\}}(x,Q^2) &= \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) F_1(x,Q^2) + \left(P^\mu - \frac{\nu}{q^2}q^\mu\right)\left(P^\nu - \frac{\nu}{q^2}q^\nu\right)\frac{F_2(x,Q^2)}{\nu} \\ W^{[\mu\nu]}(x,Q^2) &= i\epsilon^{\mu\nu\rho\sigma} q_\rho \left(\frac{S_\sigma}{\nu}(g_1(x,Q^2) + g_2(x,Q^2)) - \frac{q\cdot S P_\sigma}{\nu^2} g_2(x,Q^2)\right) \end{aligned}$$

  with $\nu = q\cdot P$, $S^2 = -M^2$, $x = Q^2/2\nu$.

  - Unpolarized: $F_1(x,Q^2)$, $F_2(x,Q^2)$,
  - Polarized: $g_1(x,Q^2)$, $g_2(x,Q^2)$.

- The same structure funtions appear in RHIC/Spin and Drell-Yang,
  - which may also provide $\langle 1\rangle_{\delta q}$ or $\langle P,S|\bar{\psi} i\gamma_5\sigma_{\mu\nu}\psi|P,S\rangle$.

---

[2] The Particle Data Group.

Moments of the structure functions are accessible on the lattice:

$$\begin{aligned} 2\int_0^1 dx x^{n-1} F_1(x,Q^2) &= \sum_{q=u,d} c_{1,n}^{(q)}(\mu^2/Q^2, g(\mu))\ \langle x^n\rangle_q(\mu) + \mathcal{O}(1/Q^2), \\ \int_0^1 dx x^{n-2} F_2(x,Q^2) &= \sum_{f=u,d} c_{2,n}^{(q)}(\mu^2/Q^2, g(\mu))\ \langle x^n\rangle_q(\mu) + \mathcal{O}(1/Q^2), \\ 2\int_0^1 dx x^{n} g_1(x,Q^2) &= \sum_{q=u,d} e_{1,n}^{(q)}(\mu^2/Q^2, g(\mu))\ \langle x^n\rangle_{\Delta q}(\mu) + \mathcal{O}(1/Q^2), \\ 2\int_0^1 dx x^{n} g_2(x,Q^2) &= \frac{1}{2}\frac{n}{n+1}\sum_{q=u,d} [e_{2,n}^{q}(\mu^2/Q^2, g(\mu))\ d_n^q(\mu) - 2e_{1,n}^{q}(\mu^2/Q^2, g(\mu))\ \langle x^n\rangle_{\Delta q}(\mu)] + \mathcal{O}(1/Q^2) \end{aligned}$$

- c_1, c_2, e_1, and e_2 are the Wilson coefficients (perturbative),
- $\langle x^n\rangle_q(\mu)$, $\langle x^n\rangle_{\Delta q}(\mu)$ and $d_n(\mu)$ are forward nucleon matrix elements of certain local operators,
- so is $\langle x^n\rangle_{\delta q}(\mu)$ which may be provided by polarized Drell-Yang and RHIC Spin.

Lattice operators:

- Unpolarized (F_1/F_2):

$$\frac{1}{2}\sum_s \langle P,S|\mathcal{O}^q_{\{\mu_1\mu_2\cdots\mu_n\}}|P,S\rangle = 2\langle x^{n-1}\rangle_q(\mu)[P_{\mu_1}P_{\mu_2}\cdots P_{\mu_n} + \cdots - (\text{trace})]$$

$$\mathcal{O}^q_{\mu_1\mu_2\cdots\mu_n} = \bar{q}\left[\left(\frac{i}{2}\right)^{n-1}\gamma_{\mu_1}\overleftrightarrow{D}_{\mu_2}\cdots\overleftrightarrow{D}_{\mu_n} - (\text{trace})\right]q$$

  On the lattice we can measure: $\langle x\rangle_q$, $\langle x^2\rangle_q$ and $\langle x^3\rangle_q$.

  - Higher moment operators mix with lower dimensional ones: operators belonging in irreducible representations of $O(4)$ transform reducibly under the lattice Hyper-cubic group.
  - Only $\langle x\rangle_q$ can be measured with $\vec{P}=0$.

- Polarized (g_1/g_2):

$$-\langle P,S|\mathcal{O}^{5q}_{\{\sigma\mu_1\mu_2\cdots\mu_n\}}|P,S\rangle = \frac{2}{n+1}\langle x^n\rangle_{\Delta q}(\mu)[S_\sigma P_{\mu_1}P_{\mu_2}\cdots P_{\mu_n} + \cdots - (\text{traces})]$$

$$\mathcal{O}^{5q}_{\sigma\mu_1\mu_2\cdots\mu_n} = \bar{q}\left[\left(\frac{i}{2}\right)^n \gamma_5\gamma_\sigma \overleftrightarrow{D}_{\mu_1}\cdots\overleftrightarrow{D}_{\mu_n} - (\text{traces})\right]q$$

$$\langle P,S|\mathcal{O}^{[5]q}_{[\sigma\{\mu_1]\mu_2\cdots\mu_n\}}|P,S\rangle = \frac{1}{n+1}d^q_n(\mu)[(S_\sigma P_{\mu_1} - S_{\mu_1}P_\sigma)P_{\mu_2}\cdots P_{\mu_n} + \cdots - (\text{traces})]$$

$$\mathcal{O}^{[5]q}_{[\sigma\mu_1]\mu_2\cdots\mu_n} = \bar{q}\left[\left(\frac{i}{2}\right)^n \gamma_5\gamma_{[\sigma} \overleftrightarrow{D}_{\mu_1]}\cdots\overleftrightarrow{D}_{\mu_n} - (\text{traces})\right]q$$

and transversity (h_1):

$$\langle P,S|\mathcal{O}^{\sigma q}_{\rho\nu\{\mu_1\mu_2\cdots\mu_n\}}|P,S\rangle = \frac{2}{m_N}\langle x^n\rangle_{\delta q}[(S_\rho P_\nu - S_\nu P_\rho)P_{\mu_1}P_{\mu_2}\cdots P_{\mu_n} + \cdots - (\text{traces})]$$

$$\mathcal{O}^{\sigma q}_{\rho\nu\mu_1\mu_2\cdots\mu_n} = \bar{q}[\left(\frac{i}{2}\right)^n \gamma_5\sigma_{\rho\nu} \overleftrightarrow{D}_{\mu_1}\cdots\overleftrightarrow{D}_{\mu_n} - (\text{traces})]q$$

  - On the lattice we can measure: $\langle 1\rangle_{\Delta q}$ (g_A), $\langle x\rangle_{\Delta q}$, $\langle x^2\rangle_{\Delta q}$, d_1, d_2, $\langle 1\rangle_{\delta q}$ and $\langle x\rangle_{\delta q}$.
  - Only $\langle 1\rangle_{\Delta q}$, $\langle x\rangle_{\Delta q}$, d_1, and $\langle 1\rangle_{\delta q}$ can be measured with $\vec{P}=0$.

On the lattice, in general, we calculate the relevant matrix elements of these currents

- with a lattice cutoff, $a^{-1} \sim$ 1-2 GeV,
- and extrapolate to the continuum, $a \rightarrow 0$,

introducing lattice renormalization, e.g.: $g_{V,A}^{\text{renormalized}} = Z_{V,A} g_{V,A}^{\text{lattice}}$.

Also, unwanted lattice artefact may result in unphysical mixing of chirally distinct operators.

DWF makes ratios such as g_A/g_V particularly easy, because:

- the chiral symmetry is almost exact, and
- maintains $Z_A = Z_V$, so that $g_A^{\text{lattice}}/g_V^{\text{lattice}}$ directly yields the renormalized value.

And other non-perturbative renormalizations are feasible too, with usually negligible unwanted mixings, and typically 10% effect at the present cuts off.

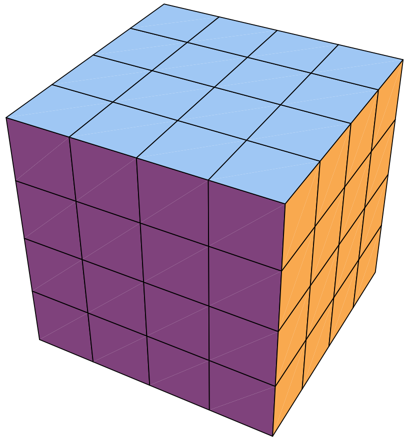
Similarly, $\langle x \rangle_{u-d}$ and $\langle x \rangle_{\Delta u - \Delta d}$ share a common renormalization, making their ratio $\langle x \rangle_{u-d}/\langle x \rangle_{\Delta u - \Delta d}$ naturally renormalized.

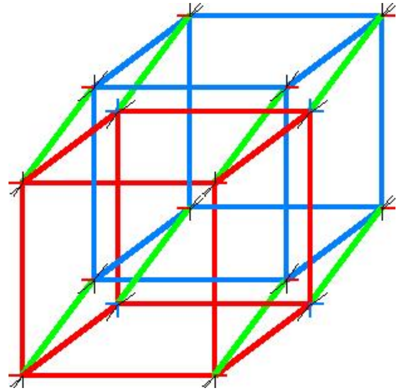
DWF is a powerful tool for calculating hadron matrix elements, such as

- meson decay constants, f_π, f_K, f_B, ...
- CP violations, B_K, ϵ'/ϵ,
- nucleon form factors and structure functions,

on the lattice: RBC had demonstrated this.

(Simplest) lattice: 4D simple hyper-cubic lattice, $L_0L_1L_2L_3$, already Euclidean





site: $s = (n_0n_1n_2n_3)$, $0 \le n_i \le L_i - 1$ $(i = 0, 1, 2, 3)$.

link: $l = (s, \mu)$, $\mu \in \{0, 1, 2, 3\}$, connects s and $s + \hat{\mu}$.

constant separation (lattice constant) a between neighboring sites.

Taking $a \to 0$ through asymptotic scalinggives exactcontinuum physics.

Dynamical variables:

quark: $q(s)$, defined on site and forms basis of fundamental (3) representation of SU(3),

gluon: $U(s, \mu) = \exp(ig \int_s^{s+\hat{\mu}} A_\mu(y)\mathrm{d}y_\mu) \in \mathrm{SU}(3)$, now a group element defined on link.

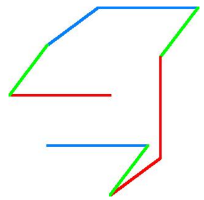
There are many other ways to define lattice (eg. random lattice) with different advantages, but the way q, U and G are defined is basically the same.

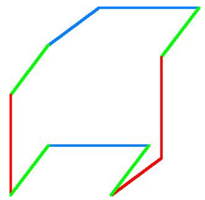
Gauge transformation: $G(s) \in \mathrm{SU}(3)$, defined on site, maps quarks and gluons

$$q(s) \mapsto G(s)q(s) \text{ and } U(s,\mu) \mapsto G(s)U(s,\mu)G(s+\hat{\mu})^{-1}.$$

Gauge invariant objects (QCD action, observables):

- Quark: $\bar{\psi}(x)U(x,\mu)U(x+\hat{\mu},\nu)...U(y-\hat{\rho},\rho)\psi(y), \mapsto \bar{\psi}(x)\underline{G^{-1}(x)G(x)}U(x,\mu)\underline{G^{-1}(x+\hat{\mu})G(x+\hat{\mu})}U(x+\hat{\mu},\nu)...U(y-\hat{\rho},\rho)\underline{G^{-1}(y)G(y)}\psi(y)$.





- Gluon, $\mathrm{Tr}[U(x,\mu)U(x+\hat{\mu},\nu)...U(x-\hat{\rho},\rho)] \mapsto \mathrm{Tr}[\underline{G(x)}U(x,\mu)\underline{G^{-1}(x+\hat{\mu})G(x+\hat{\mu})}U(x+\hat{\mu},\nu)...U(x-\hat{\rho},\rho)\underline{G^{-1}(x)}]$.

Action: $S_{\mathrm{QCD}}[U,q,\bar{q}] = S_{\mathrm{gluon}}[U] + S_{\mathrm{quark}}[U,q,\bar{q}]$, must respect gauge invariance:

gluon part: such as $S_{\mathrm{gluon}}[U] = \frac{6}{g^2}\sum_s \sum_{\mu<\nu} \Box(s,\mu,\nu)$, gives $-\frac{1}{2}\mathrm{Tr}G^{\mu\nu}G_{\mu\nu}$ as $a \to 0$ and $g \to 0$,

- where the plaquette, $\Box(s,\mu,\nu) = 1 - \frac{1}{3}\mathrm{ReTr}U(s,\mu)U(s+\hat{\mu},\nu)U(s+\hat{\nu},\mu)^{-1}U(s,\nu)^{-1}$.

quark part: $S_{\mathrm{quark}}[U,q,\bar{q}] = \sum_{s,s'} \bar{q}(s)M[U](s,s')q(s')$, which should give $\bar{q}(i\gamma^\mu D_\mu - m)q$ as $a \to 0$ and $g \to 0$,

- with $M[U](s,s')$ describing quark propagation between sites s and s'.

Expectation values of any gauge-invariant observable: $\langle O\rangle = N^{-1}\int[\mathrm{d}U][\mathrm{d}q][\mathrm{d}\bar{q}]O[U,q,\bar{q}]\exp(-S_{\mathrm{QCD}}[U,q,\bar{q}])$,
or by integrating over the quark Grassmann variables: $N'^{-1}\int[\mathrm{d}U](\det M[U])\exp(-S_{\mathrm{gluon}}[U])$.
It is often convenient to use effective action: $\tilde{S}[U] = S_{\mathrm{gluon}}[U] - \mathrm{Tr}\log M[U]$.

Finite lattice and compact SU(3) assures finite $\langle O\rangle$.

Continuum limit is well defined because of the asymptotic freedom: consider an observable O with mass dimension,

- the expectation value is described as $\langle O\rangle = a^{-1}f(g)$ with some dimensionless function $f(g)$ of dimensionless coupling g.

- Renormalizability of the theory means the cutoff dependence should vanish $\dfrac{d\langle O\rangle}{da} \to 0$ as $a \to 0$, or

$$f(g) - f'(g)\left(a\frac{dg}{da}\right) = \beta(g)f'(g) + f(g) \to 0.$$

- This ($df/f = -dg/\beta$) is easily solved to give: $\langle O\rangle a \propto \exp\left(-\int^g \dfrac{dh}{\beta(h)}\right)$, or

$$\langle O\rangle a \propto (g^2 b_0)^{-\frac{b_1}{2b_0^2}} e^{-\frac{1}{2b_0 g^2}}[1+O(g^2)],$$

where $\beta(g) \equiv -a\dfrac{dg}{da} = -b_0 g^3 - b_1 g^5 + O(g^7)$ is perturbatively well known.

Chiral symmetry:

- invariance under global $U(N_f)$ transformations, $q \mapsto \exp(i\theta)q$, $\exp(i\theta'\gamma_5)q$, $\exp(i\alpha^a \frac{\lambda^a}{2})q$ and $\exp(i\beta^a \frac{\lambda^a}{2}\gamma_5)q$.
- Should be preserved in the absence of $m\bar{q}q$, like $U(N_f)_L \times U(N_f)_R = SU(N_f)_V \times SU(N_f)_A \times U(1)_V \times U(1)_A$.
- In fact spontaneously broken for light normal quarks, $m_u \sim m_d \sim 0$, $\langle \bar{u}u + \bar{d}d \rangle \neq 0$.
- Important for Nambu-Goldstone pion, PCAC, etc, $m_\pi^2 f_\pi^2 = m_q \langle \bar{q}q \rangle$.

However, very hard to maintain on regular lattices.

Naive lattice fermion action, with $M_{xy} = \frac{1}{2} a^{D-1} \sum_\mu \gamma_\mu [\delta_{x+\hat{\mu},y} - \delta_{x-\hat{\mu},y}]$, leads to a propagator $\Delta(p) = a[\gamma_\mu \sin(p_\mu a)]^{-1}$, which has 2^D poles at $p_\mu = 0$ or π/a: for $D = 4$ there appear $2^4 = 16$ flavors instead of one.

Shifting of one component of p_μ, such as $\tilde{p}_\mu = p_\mu - \pi/a$, acts like

$$\gamma_\mu \sin(p_\mu a) = -\gamma_\mu \sin(\tilde{p}_\mu a)$$

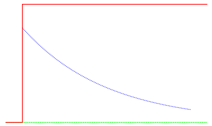
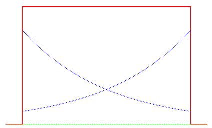
so the chirality $\pm$ states are paired.

Nielsen and Ninomiya theorem: doubling inevitable (chirality $\pm$ states are paired) for a regular lattice and local, hermitian, and translationally invariant action.

Kaplan's Domain-wall fermions[3]: introduce a fifth dimension, s, maximally break the 5D chiral symmetry, so that a 4D chiral symmetry survives. Define a 5D Dirac operator: $D = \gamma_\mu \partial_\mu + \gamma_5 \partial_s + m(s)$,

- with a monotonic $m(s)$ with $m(s=0)=0$, a 4D chiral modes emerge: $\psi_\pm(x,s) = u_p(x)\phi_\pm(s)\chi_\pm$.
- With 4D Dirac plane wave u_p and γ_5 eigenstate, $\gamma_5\chi_\pm = \pm\chi_\pm$, the s-dependence is dictated by $[\pm\partial_s + m(s)]\phi(s) = 0$, which leads to

$$\phi(s) \propto \exp[\mp \int_0^s ds' m(s')],$$

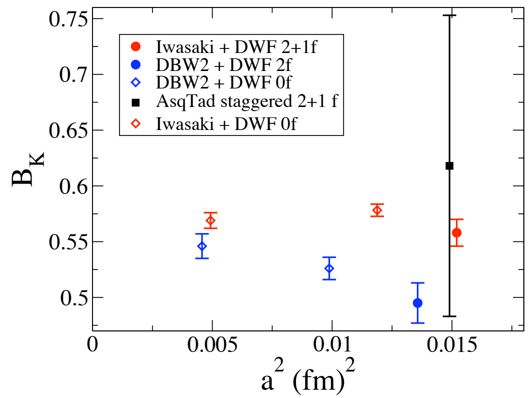
  - pinned at the $s=0$ wall, and exponentially decay to $\pm s$ direction  .
  - Very nice, but on a finite lattice, two walls, with a pair of $\pm$ chiralities  mix.

- Shamir[4]: no problem for a vector theory like QCD. Two-wall mixing exponentially suppressed, described by $m_{\rm res}$.

RIKEN-BNL-Columbia (RBC) Collaboration proved DWF works very well for QCD:

- light hadron mass spectrum,
- electroweak transitions among light hadrons (such as B_K in the right),

unlike conventional Wilson and staggered fermions.



[3] D.B. Kaplan, Phys. Lett. B288, 342 (1992), hep=lat/9206013.
[4] Y. Shamir, Nucl. Phys. B406, 90 (1993), hep-lat/9303005; V. Furman and Y. Shamir, Nucl. Phys. B439, 54 (1995), hep-lat/9405004; and references cited therein.

QCDSP and QCDOC computers: dedicated for lattice QCD calculations.

QCDSP: completed in 1998, 600 (RBRC) and 400 (Columbia) GFlops configurations

- based on commercial DSP
- assisted by custom designed 4D hypercubic nearest-neighbor communication
- 10$ per MFlops

Demonstrated the use of DWF in (quenched) lattice QCD

- Chiral and flavor symmetries and associated ease in non-perturbative renormalizations,
- hadron spectroscopy: masses and decay constants,
- hadron matrix elements: B_K, ϵ'/ϵ, nucleon form factors and structure functions.

QCDOC: complete in 2005, 10 TFLops configurations in RBRC, BNL and Edinburgh.

- based on system on a chip technology,
- a QCDSP card was shrunk to be a QCDOC chip, with custom-designed 6D hypercubic communicaitons,
- 1$ per MFlops.

Used for realistic (2+1)-flavor dynamical DWF lattice QCD. And evolved into BG/L.

Our formulation follows the standard one,

- Two-point function: $G_{N}(t) = \mathrm{Tr}[(1+\gamma_t)\sum_{\vec{x}}\langle TB_1(x)B_1(0)\rangle]$, using $B_1 = \epsilon_{abc}(u_a^T C\gamma_5 d_b)u_c$ for proton,
- Three-point functions,
  - vector: $G_{V}^{u,d}(t,t') = \mathrm{Tr}[(1+\gamma_t)\sum_{\vec{x}'}\sum_{\vec{x}}\langle TB_1(x')V_t^{u,d}(x)B_1(0)\rangle]$,
  - axial: $G_{A}^{u,d}(t,t') = \frac{1}{3}\sum_{i=x,y,z}\mathrm{Tr}[(1+\gamma_t)\gamma_i\gamma_5\sum_{\vec{x}'}\sum_{\vec{x}}\langle TB_1(x')A_i^{u,d}(x)B_1(0)\rangle]$.

  with fixed $t' = t_{\mathrm{sink}} - t_{\mathrm{source}}$ and $t < t'$.
- From the lattice estimate
$$g_{\Gamma}^{\mathrm{lattice}} = \frac{G_{\Gamma}^{u}(t,t') - G_{\Gamma}^{d}(t,t')}{G_{N}(t)},$$
  with $\Gamma = V$ or A, the renormalized value
$$g_{\Gamma}^{\mathrm{ren}} = Z_{\Gamma} g_{\Gamma}^{\mathrm{lattice}},$$
  is obtained.
- Non-perturbative renormalizations, defined by
$$[\bar{u}\Gamma d]_{\mathrm{ren}} = Z_{\Gamma}[\bar{u}\Gamma d]_0,$$
  satisfies $Z_{A} = Z_{V}$ well, so that
$$\left(\frac{g_A}{g_V}\right)^{\mathrm{ren}} = \left(\frac{G_{A}^{u}(t,t') - G_{A}^{d}(t,t')}{G_{V}^{u}(t,t') - G_{V}^{d}(t,t')}\right)^{\mathrm{lattice}}.$$

g_{A} is also described as $\Delta u - \Delta d$.

Renormalization: $\mathcal{O}^{\rm ren} = Z_{\mathcal{O}}(a\mu)\mathcal{O}^{\rm lat}(a)$,

- lattice complications: operator mixing from broken Lorentz or chiral symmetry,
- NPR is required when mixing with lower dimensional operator occurs.

We calculate $Z_{\mathcal{O}}(a\mu)$ non-perturbatively in RI/MOM scheme[5] with perturbative matching to $\overline{\rm MS}$.

- compute off-shell matrix element of the operator, $\mathcal{O}$, in Landau gauge,
- impose a MOM scheme condition $\mathrm{Tr}\, V_{\mathcal{O}}(p^2)\Gamma\big|_{p^2=\mu^2} \dfrac{Z_{\mathcal{O}}}{Z_q} = 1$,
  - $V_{\mathcal{O}}(p^2)$ is the relevant amputated vertex,
  - Γ is an appropriate projector,
- extrapolate to the chiral limit, defining the RI scheme,
- in an appropriate window, $\Lambda_{\rm QCD} \ll \mu^2 \ll a^{-1}$, a scale invariant

$$Z_{\rm rgi} = \frac{Z(\mu^2)}{C(\mu^2)}$$

  is obtained, with the operator running $C(\mu^2)$ in the continuum perturbation theory.
- Now we can perturbatively match to e.g. $\overline{\rm MS}$.

Works nicely with DWF.

---

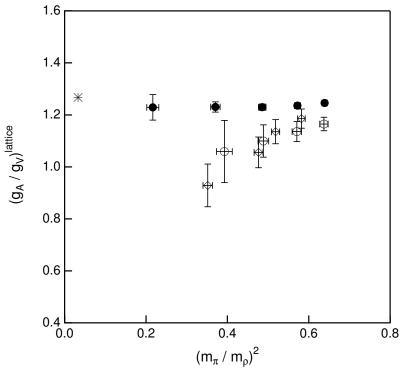
[5] Martinelli et. al, Nucl. Phys. B455, 81 (1995).

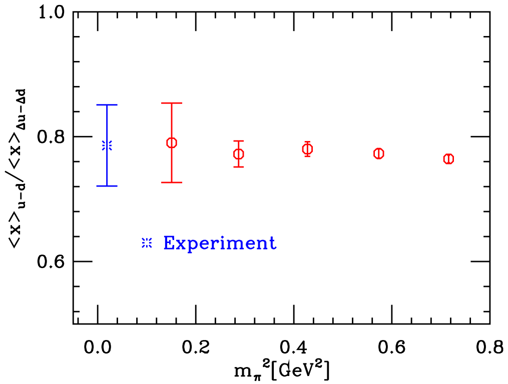
Rectangular gauge actions such as Iwasaki and DBW2, $S_G = \beta[c_0 \sum W_{1,1} + c_1 \sum W_{1,2}]$, with $c_0 + 8c_1 = 1$, help both:

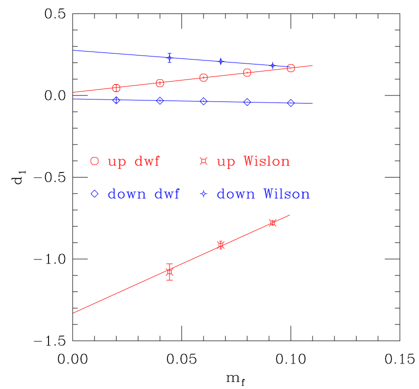
- good chiral behavior, *i.e.* close enough to the continuum, and
- sufficiently large volume to contain a nucleon.

With quenched DBW2 calculations RBC had demonstrated[6]:

- $(g_A/g_V)^{\rm lattice} = (g_A/g_V)^{\rm ren}$ is strongly volume-dependent.
  - It is not appreciably quark-mass dependent once the volume is sufficiently large,
  - $g_A/g_V = 1.212(27)_{\rm stat.}(24)_{\rm syst.}$, compared with the experiment of 1.2695(29).







- NPR for structure function moments is well-behaved.
- Both $\langle x\rangle_{u-d}$ and $\langle x\rangle_{\Delta u - \Delta d}$ overshoot the experiments by about 50%,
  - but their ratio, again naturally renormalized, agree well with the experiment
  - and again without much appreciable quark-mass dependence.
- $\langle 1\rangle_{\delta u - \delta d} = 1.19(3)$, $\overline{\rm MS}$ (2 GeV) 2-loop running.
- d_1, though not renormalized yet, appears small in the chiral limit.

[6] S. Sasaki, K. Orginos, SO, and T. Blum, Phys.Rev.D68:054509, 2003 (hep-lat/0306007); K. Orginos, T. Blum and SO, Phys.Rev.D73:094503, 2006 (hep-lat/0505024).

Here we report on two dynamical DWF ensembles.

1. RBC 2-flavor DBW2+DWF dynamical calculations:
   - $a^{-1} = 1.7$ GeV, $m_{\rm res} = 0.00137(5)$
   - $m_{\rm sea} = 0.04$, 0.03, and 0.02,
   - $m_\pi = 700$, 610, and 490 MeV; $m_N = 1.5$, 1.4, and 1.3 GeV (a few % errors),
   - single volume, $16^3 \times 32 \times 12$,
   - about 220 configurations at each $m_{\rm sea}$ value,

   Analysis is almost complete except NPR.

2. RBC/UKQCD (2+1)-flavor, Iwasaki+DWF dynamical calculations:
   - $a^{-1} = 1.6$ GeV, $m_{\rm res} = 0.003$,
   - $m_{\rm strange} = 0.04$ and $m_{\rm up,down} = 0.03$, 0.02, 0.01 and 0.005,
   - $m_\pi = 620$, 520, 390 and 310 MeV; $m_N = 1.4$, 1.3, 1.2 and 1.1 GeV,
   - two volumes,
     - $16^3 \times 32 \times 16$, 2 fm across, ensemble production complete, UKQCD analyses ongoing, and
     - $24^3 \times 64 \times 16$, 3 fm across, ensemble production ongoing, RBC analyses ongoing (24-30 configurations).

   Preliminary analysis for the larger volume.

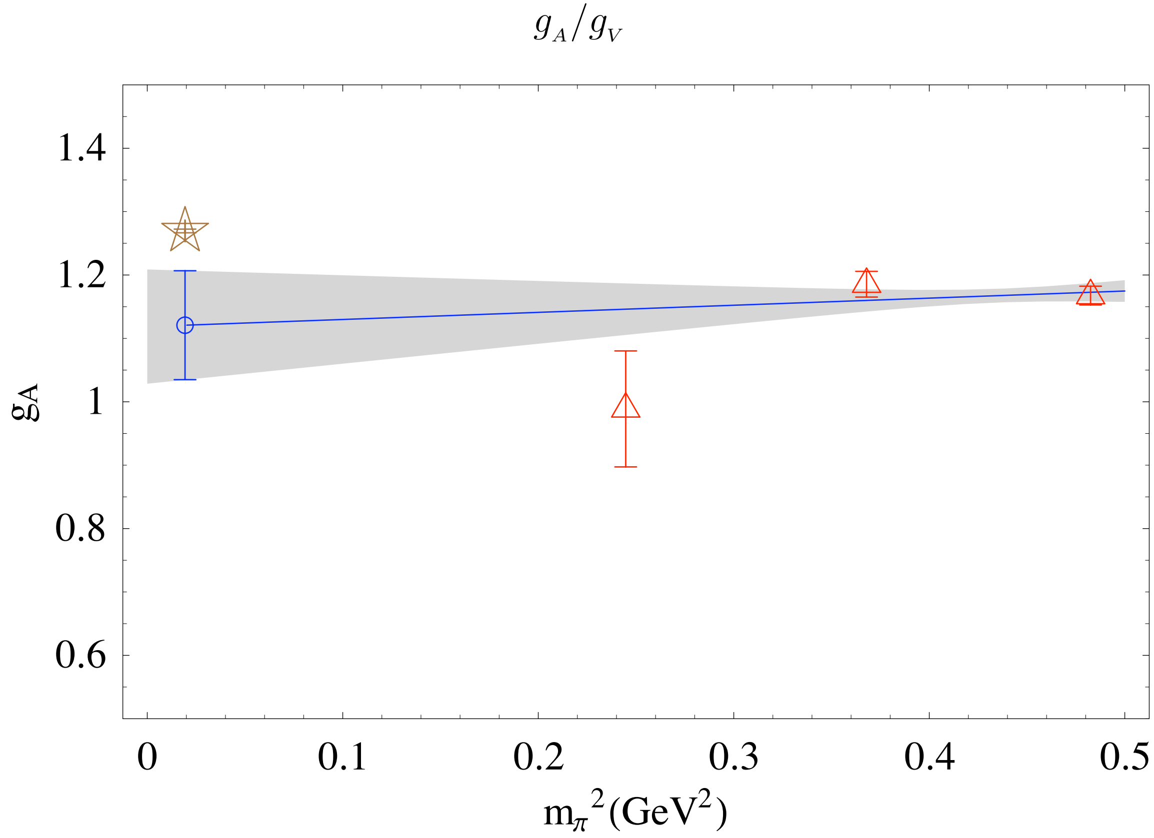
Observables: those not requiring finite momentum transfer

- ratios g_A/g_V and $\langle x\rangle_{u-d}/\langle x\rangle_{\Delta u-\Delta d}$, naturally renormalized,
- $g_{V,T,A,P}(q^2)$, $\langle x\rangle_{u-d}$, $\langle x\rangle_{\Delta u-\Delta d}$, $\langle 1\rangle_{\delta u-\delta d}$, all fully non-perturbatively renormalized,
- d_1, yet to be renormalized but still interesting.

bare values of the others have been calculated but are waiting for renormalization.

RBC 2-flavor DBW2+DWF dynamical: $a^{-1} = 1.7$ GeV, $16^3 \times 32 \times 12$, $m_{\rm res} = 0.00137(5)$, about 220 configurations,

- $m_{\rm sea} = 0.04$, 0.03, and 0.02; $m_\pi = 700$, 610, and 490 MeV; $m_N = 1.5$, 1.4, and 1.3 GeV (a few % errors),



$g_A/g_V = 1.12(9)$ compares well with experiment, 1.2695(29), with mild quark-mass dependence.
Or beginning to see the smallness of the box at $m_\pi L \sim 5$?

Momentum transfer dependence of isovector vector- and axialvector-current form factors:

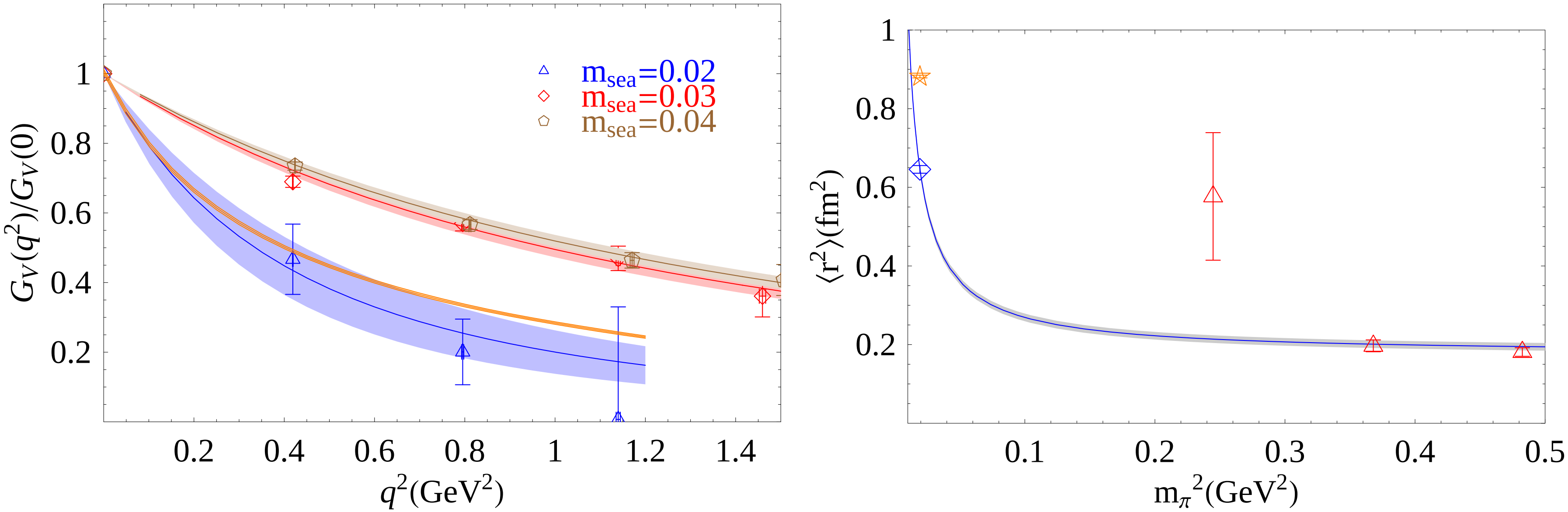
	$m_f a$	$n^2 = 0$	$n^2 = 1$	$n^2 = 2$	$n^2 = 3$	$n^2 = 4$
	0.02	1.27(3)	0.59(12)	0.25(12)	N/A	N/A
G_V	0.03	1.291(3)	0.89(2)	0.74(3)	0.61(5)	0.47(8)
	0.04	1.2858(13)	0.945(14)	0.728(17)	0.60(3)	0.52(6)
	0.02		1.5(4)	0.8(5)	N/A	N/A
G_T	0.03		2.60(10)	2.02(9)	1.56(13)	1.00(16)
	0.04		2.74(8)	2.01(6)	1.68(9)	1.08(12)
	0.02	1.27(11)	0.98(14)	0.67(16)	N/A	N/A
G_A	0.03	1.53(3)	1.17(3)	1.01(3)	0.88(6)	0.63(9)
	0.04	1.50(2)	1.21(2)	1.02(3)	0.87(4)	0.78(7)
	0.02		8.5(2.3)	4.8(1.4)	N/A	N/A
G_P	0.03		9.1(6)	6.0(3)	5.0(4)	1.8(7)
	0.04		9.9(4)	6.2(3)	4.6(3)	3.1(5)

Induced form factors (G_T and G_P) made dimensionless with $2m_N$ normalization.
Share common renormalization given by $Z_V = 1/G_V(0)$.

RBC 2-flavor DBW2+DWF dynamical: $a^{-1} = 1.7$ GeV, $16^3 \times 32 \times 12$, $m_{\rm res} = 0.00137(5)$, about 220 configurations,

- $m_{\rm sea} = 0.04$, 0.03, and 0.02; $m_\pi = 700$, 610, and 490 MeV; $m_N = 1.5$, 1.4, and 1.3 GeV (a few % errors),

$G_V(q^2)$: $F_1^p - F_1^n$



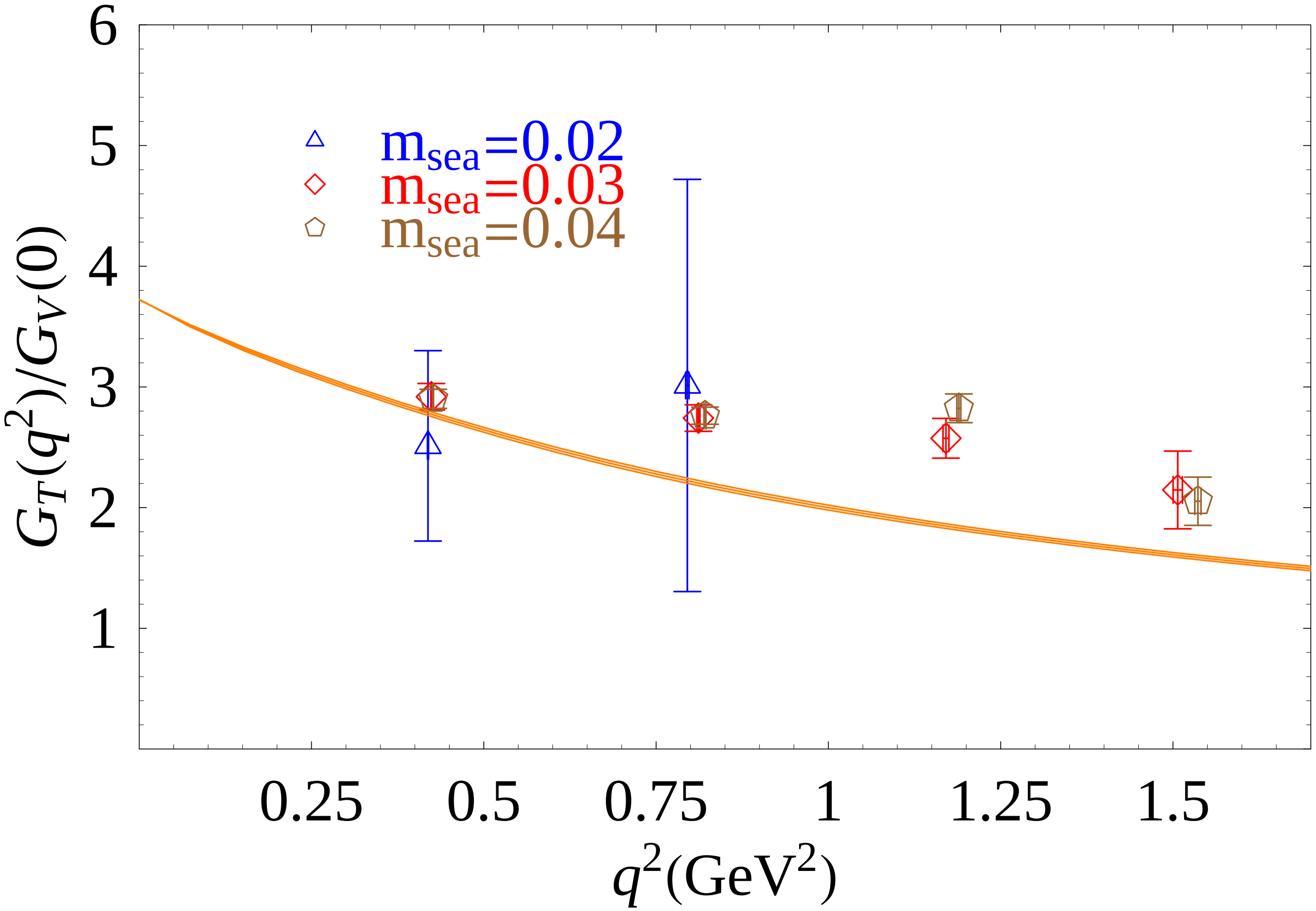
fit well by conventional dipole $(1 + q^2/M^2)^{-2}$, with the conventional value of $M^2 = 0.71$ GeV^2 allowing extraction of rms radii, $\langle r^2 \rangle_{\rm Dirac}$, which shows encouraging trend toward experiment.

Or beginning to see the smallness of the box at $m_\pi L \sim 5$?

RBC 2-flavor DBW2+DWF dynamical: $a^{-1} = 1.7$ GeV, $16^3 \times 32 \times 12$, $m_{\rm res} = 0.00137(5)$, about 220 configurations,

- $m_{\rm sea} = 0.04$, 0.03, and 0.02; $m_\pi = 700$, 610, and 490 MeV; $m_N = 1.5$, 1.4, and 1.3 GeV (a few % errors),

$G_T(q^2)$, $F_2^p - F_2^n$



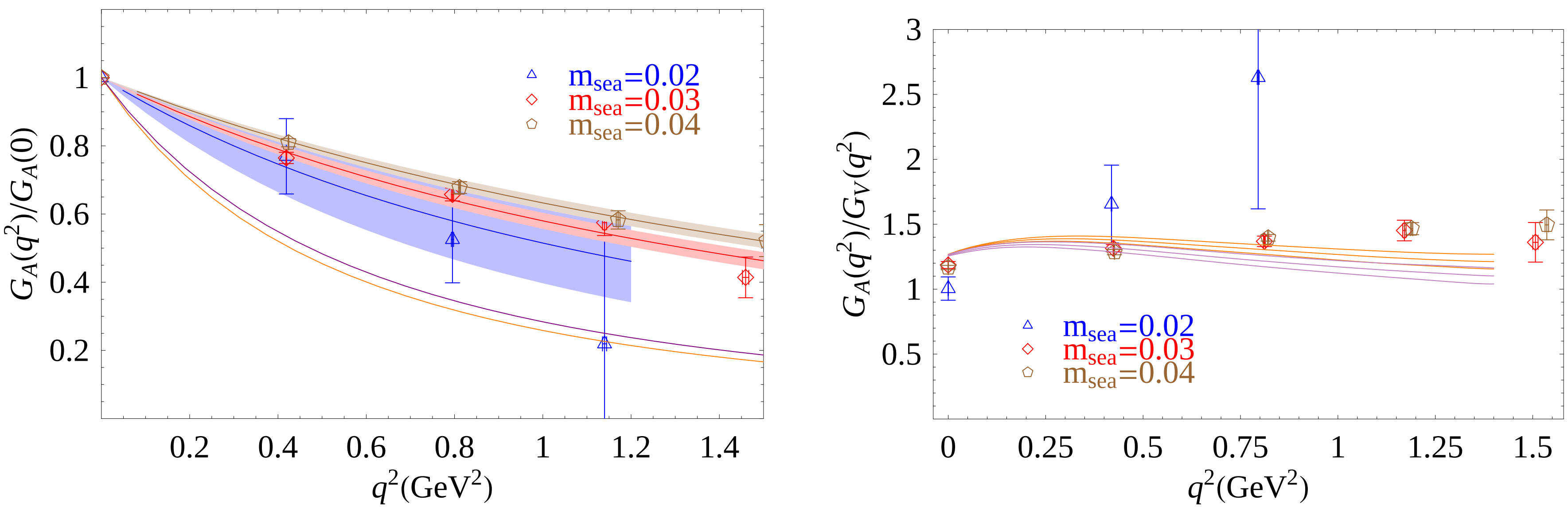
dipole fit works

reasonable agreement with experimental magnetic moment.

RBC 2-flavor DBW2+DWF dynamical: $a^{-1} = 1.7$ GeV, $16^3 \times 32 \times 12$, $m_{\rm res} = 0.00137(5)$, about 220 configurations,

- $m_{\rm sea} = 0.04$, 0.03, and 0.02; $m_\pi = 700$, 610, and 490 MeV; $m_N = 1.5$, 1.4, and 1.3 GeV (a few % errors),

$G_A(q^2)$

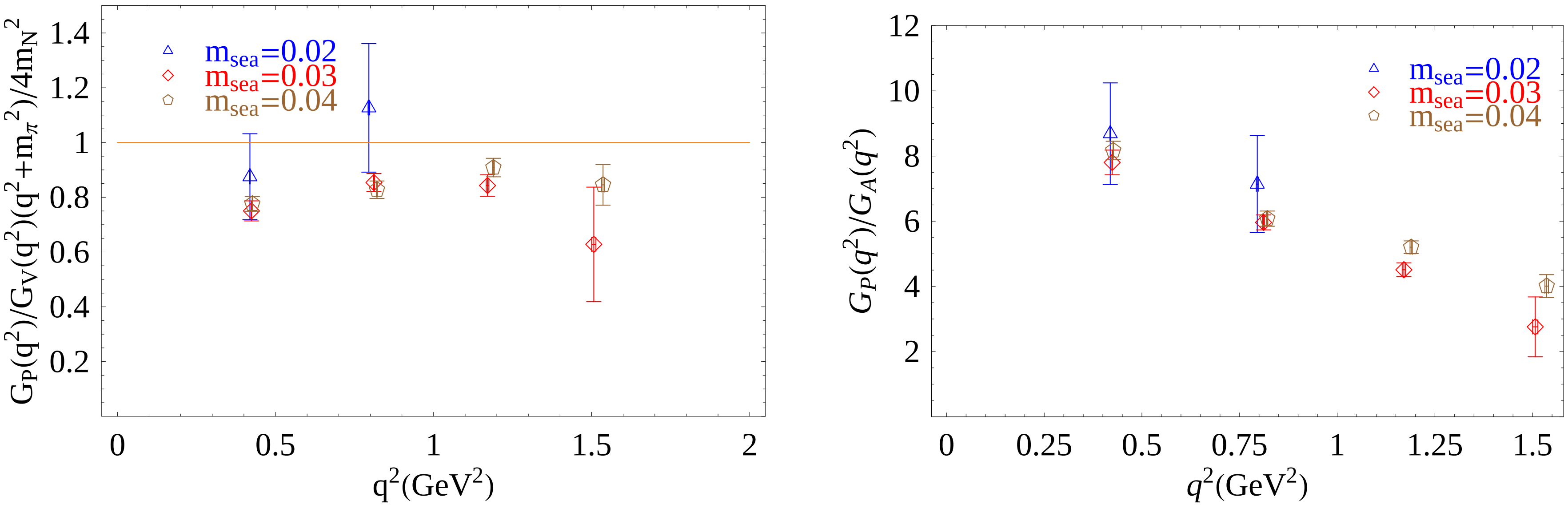


lattice results follow dipole form, but overshoot dipole form with experimental dipole mass, 1.0-1.1 GeV^2, renormalized values (right) follow the experiments well.

RBC 2-flavor DBW2+DWF dynamical: $a^{-1} = 1.7$ GeV, $16^3 \times 32 \times 12$, $m_{\rm res} = 0.00137(5)$, about 220 configurations,

- $m_{\rm sea} = 0.04$, 0.03, and 0.02; $m_\pi = 700$, 610, and 490 MeV; $m_N = 1.5$, 1.4, and 1.3 GeV (a few % errors),

$G_P(q^2)$



comparison with pion-pole dominance (left), G_P normalized by $2m_N$ (right),

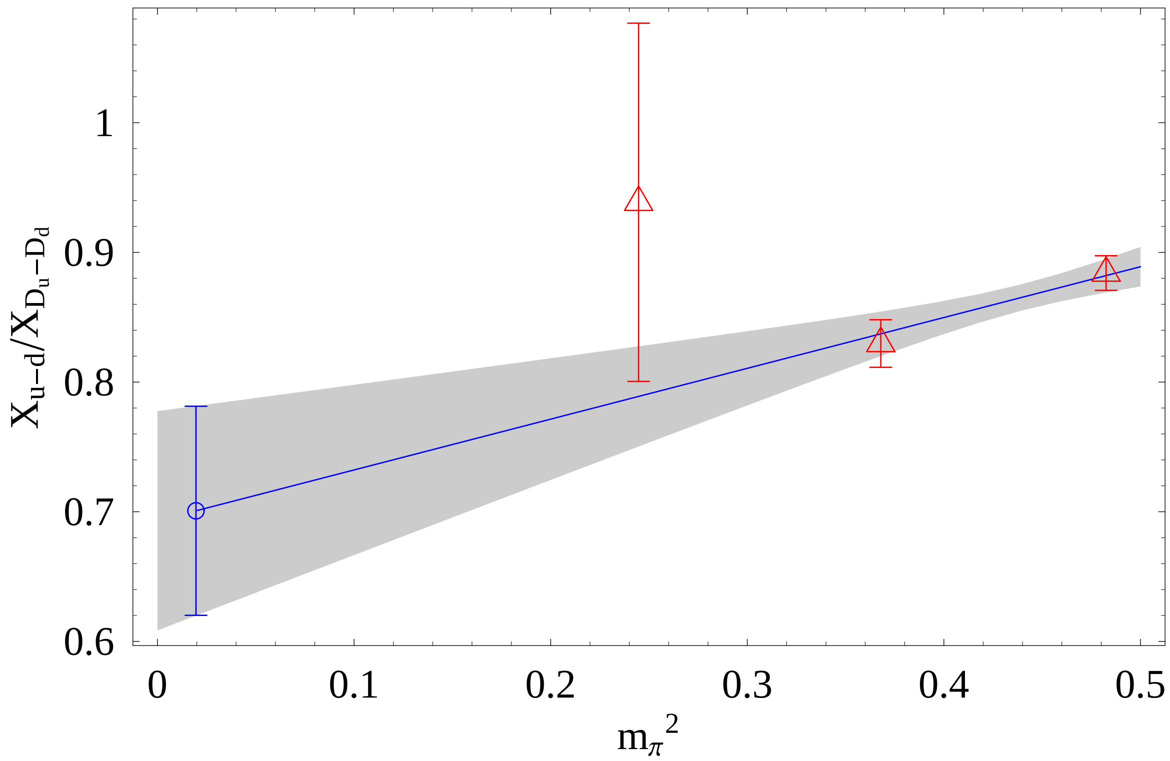
$g_{\pi NN}$ extrapolates to 12.0(9),

Goldberger-Treiman relation satisfied below lowest lattice momentum transfer.

RBC 2-flavor DBW2+DWF dynamical: $a^{-1} = 1.7$ GeV, $16^3 \times 32 \times 12$, $m_{\rm res} = 0.00137(5)$, about 220 configurations,

- $m_{\rm sea} = 0.04$, 0.03, and 0.02; $m_\pi = 700$, 610, and 490 MeV; $m_N = 1.5$, 1.4, and 1.3 GeV (a few % errors),

$\langle x \rangle_{u-d} / \langle x \rangle_{\Delta u - \Delta d}$

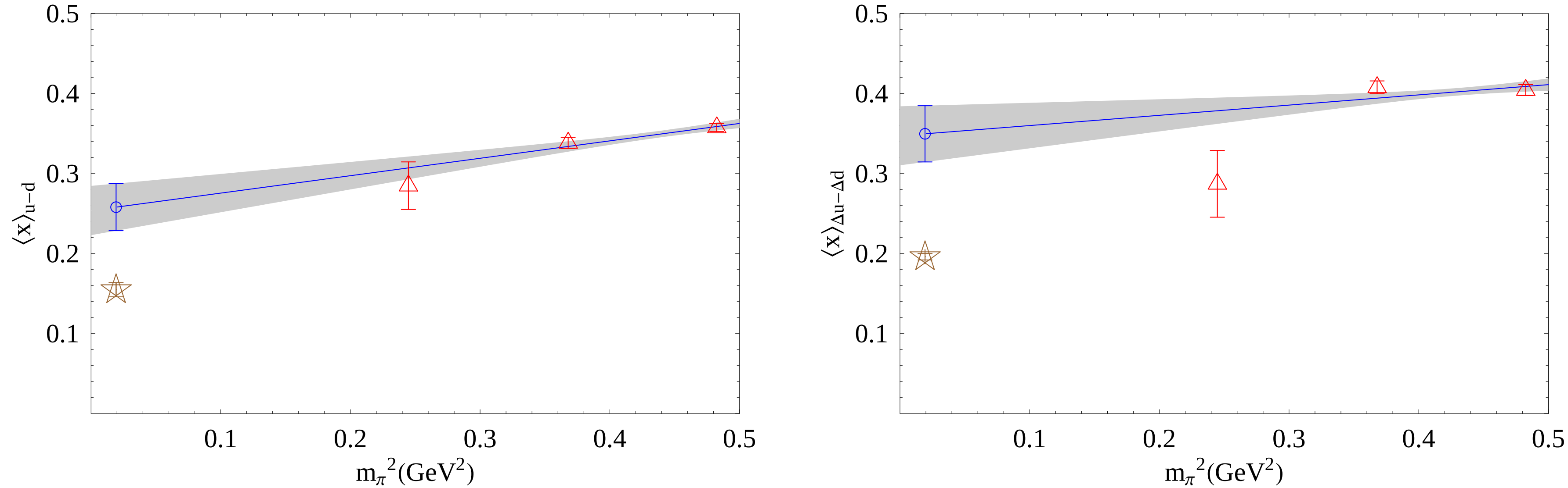


extrapolation to physical pion mass, 0.70(8), compares well with experiment, 0.79(3)
mild quark-mass dependence.

RBC 2-flavor DBW2+DWF dynamical: $a^{-1} = 1.7$ GeV, $16^3 \times 32 \times 12$, $m_{\rm res} = 0.00137(5)$, about 220 configurations,

- $m_{\rm sea} = 0.04$, 0.03, and 0.02; $m_\pi = 700$, 610, and 490 MeV; $m_N = 1.5$, 1.4, and 1.3 GeV (a few % errors),

Momentum fraction $\langle x \rangle_{u-d}$ and helicity fraction $\langle x \rangle_{\Delta u - \Delta d}$

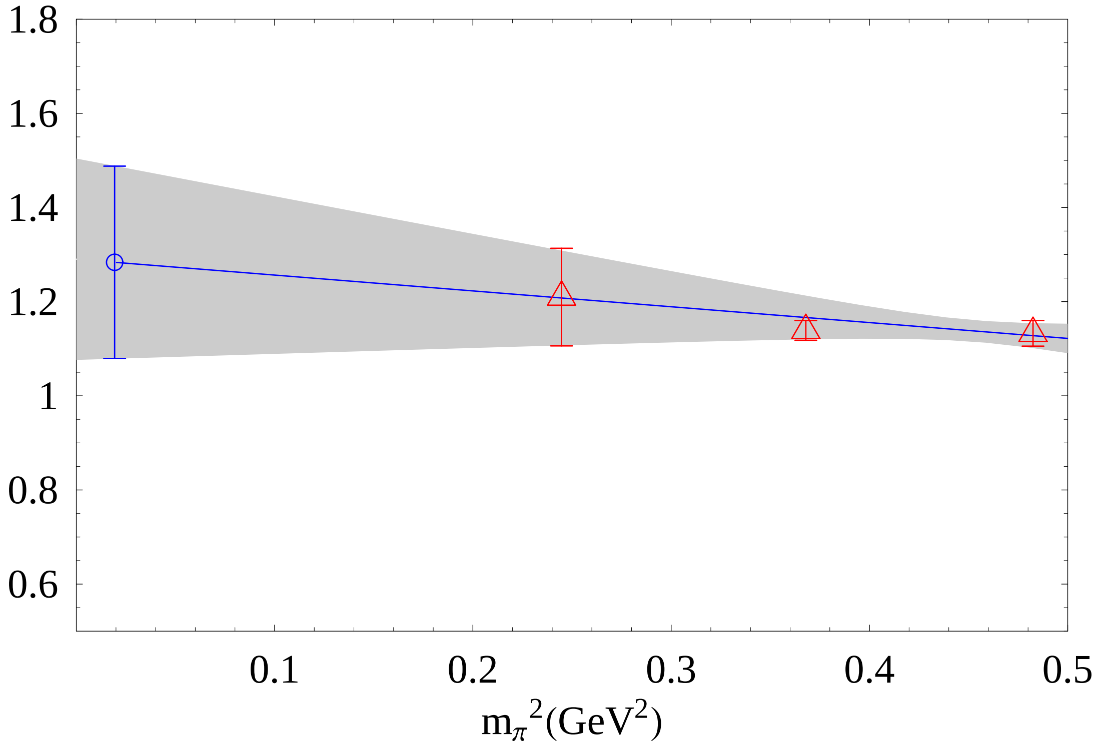


Fully non-perturbatively renormalized and run to $\overline{\rm MS}$ 2 GeV: both fractions overshoot experiments.
Lightest points trend toward experiments: encouraging, but need confirmation with full dynamical calculation.
Or beginning to see the smallness of the box at $m_\pi L \sim 5$?

RBC 2-flavor DBW2+DWF dynamical: $a^{-1} = 1.7$ GeV, $16^3 \times 32 \times 12$, $m_{\rm res} = 0.00137(5)$, about 220 configurations,

- $m_{\rm sea} = 0.04$, 0.03, and 0.02; $m_\pi = 700$, 610, and 490 MeV; $m_N = 1.5$, 1.4, and 1.3 GeV (a few % errors),

transversity $\langle 1 \rangle_{\delta u - \delta d}$

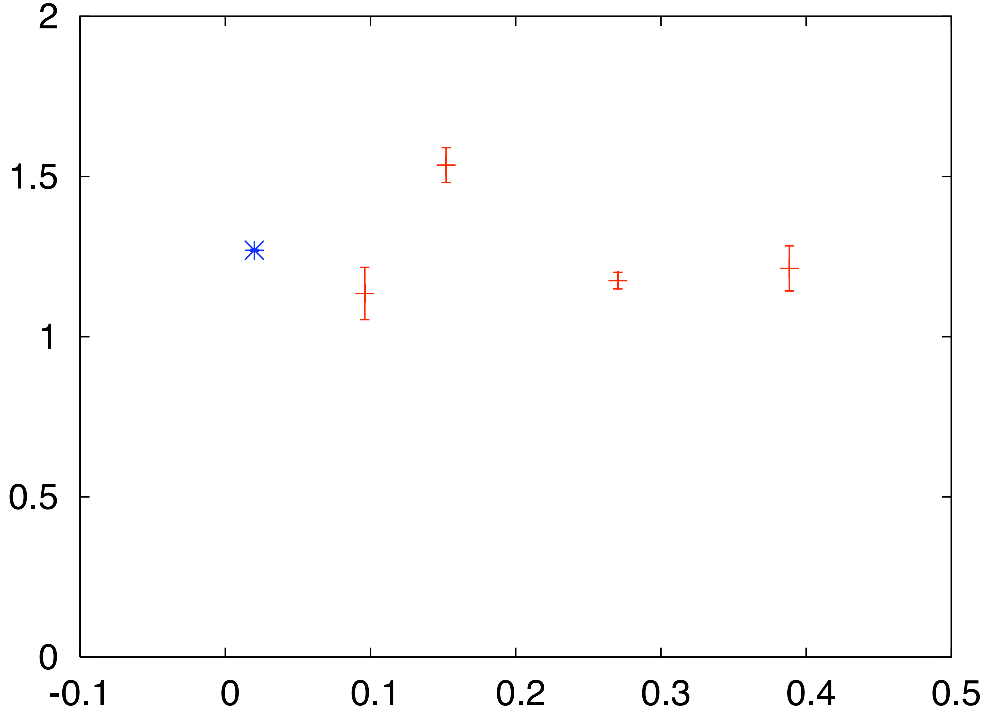


fully non-perturbatively renormalized (~ 10 %),
extrapolation of 1.28(20) awaits experiment.

RBC/UKQCD (2+1)-flavor, Iwasaki+DWF dynamical, $a^{-1} = 1.6$ GeV, $m_{\rm res} = 0.003$, $m_{\rm strange} = 0.04$,

- $m_{\rm up,down} = 0.03, 0.02, 0.01$ and 0.005; $m_\pi = 620, 520, 390$ and 310 MeV; $m_N = 1.4, 1.3, 1.2$ and 1.1 GeV,
- larger of the two volumes, $24^3 \times 64 \times 16$, 3 fm across, ongoing, preliminary analyses,

g_A/g_V

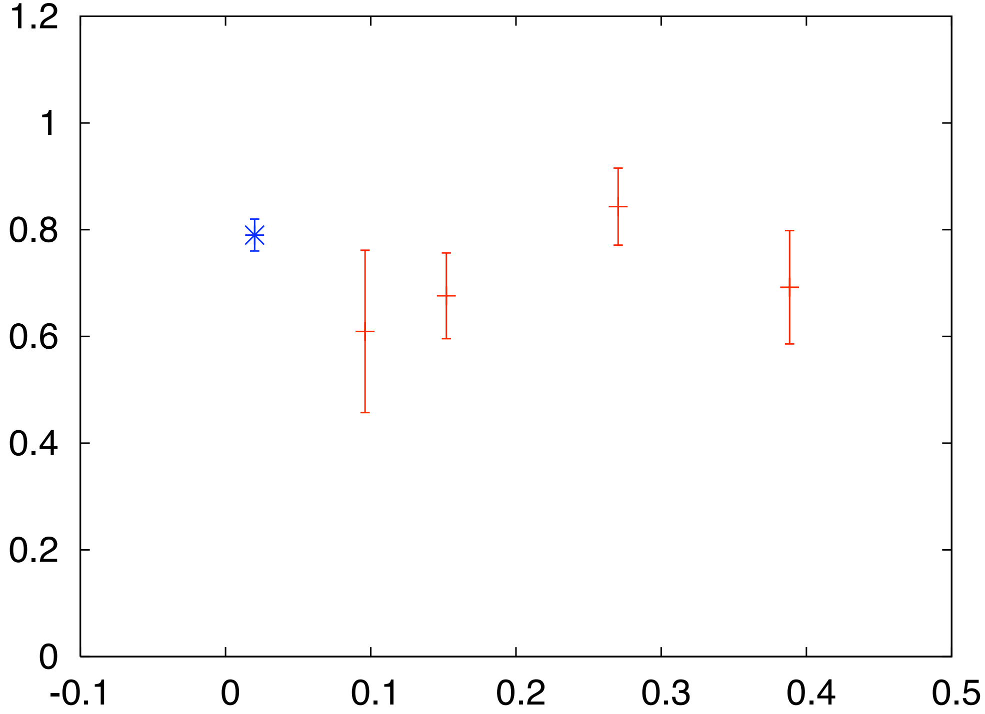


consistent with experiment, mild quark-mass dependence.
Or beginning to see the smallness of the box at $m_\pi L \sim 5$?

RBC/UKQCD (2+1)-flavor, Iwasaki+DWF dynamical, $a^{-1} = 1.6$ GeV, $m_{\rm res} = 0.003$, $m_{\rm strange} = 0.04$,

- $m_{\rm up,down} = 0.03$, 0.02, 0.01 and 0.005; $m_\pi = 620$, 520, 390 and 310 MeV; $m_N = 1.4$, 1.3, 1.2 and 1.1 GeV,
- larger of the two volumes, $24^3 \times 64 \times 16$, 3 fm across, ongoing, preliminary analyses,

$\langle x \rangle_{u-d} / \langle x \rangle_{\Delta u - \Delta d}$



consistent with experiment, mild quark-mass dependence.

Absolute values seem to have improved, trending to the experimental values, but yet to be renormalized (typically 10% or less effect at these cuts off).

Conclusions:

1. RBC 2-flavor DBW2+DWF dynamical calculations are complete with full NPR:
   - $a^{-1} = 1.7$ GeV, $(2\text{fm})^3$ box, $m_\pi \sim 0.7$, 0.6 and 0.5 GeV.
   - Ratios, g_A/g_V and $\langle x\rangle_{u-d}/\langle x\rangle_{\Delta u-\Delta d}$, in favorable comparison with experiment:
     - $g_A/g_V = 1.12(9)$, $\langle x\rangle_{u-d}/\langle x\rangle_{\Delta u-\Delta d} = 0.70(8)$, with mild quark-mass dependence.
   - Reasonable momentum-transfer dependence in form factors is seen.
   - Fully renormalized transversity $\langle 1\rangle_{\delta u-\delta d}$ extrapolates to a value 1.28(20).
   - Lightest point often trend toward experiments; Is lightest $m_\pi L \sim 5$ a problem?
     - No phenomenological model particularly favored: often simple linear extrapolation works well.

2. RBC/UKQCD (2+1)-flavor, Iwasaki+DWF dynamical calculations are ongoing,
   - $a^{-1} = 1.6$ GeV, $(3\text{fm})^3$ box, $m_\pi \sim 0.6$, 0.5, 0.4 and 0.3 GeV, though preliminary at 1/2 or 1/3 of planned statistics.
   - Ratios, g_A/g_V and $\langle x\rangle_{u-d}/\langle x\rangle_{\Delta u-\Delta d}$, consistent with experiment, mild quark-mass dependence.
   - Momentum dependences to come soon, by Lattice 2007 next month.
   - NPR to come soon, by Lattice 2007 next month.
   - Lightest point shows an encouraging trend toward experiments; Is lightest $m_\pi L \sim 5$ a problem?

3. RBC/UKQCD (2+1)-flavor, Iwasaki+DWF, $a^{-1} = 2.1$ GeV ensemble generation started:
   - same large volume,
   - lighter up/down, as light as 1/7 strange mass.

4. Spawned BG/L and BG/P, and design of BG/Q is starting.