Masses and semileptonic decays of doubly heavy baryons in a nonrelativistic quark model

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Outline of the talk

- Quark-quark interaction
  - Three-body Hamiltonian
  - Quark-quark potential
- Baryon wave function
- Variational ansatz
- Results for spectra
- Semileptonic decay
- Infinite heavy quark mass limit

Baryons studied

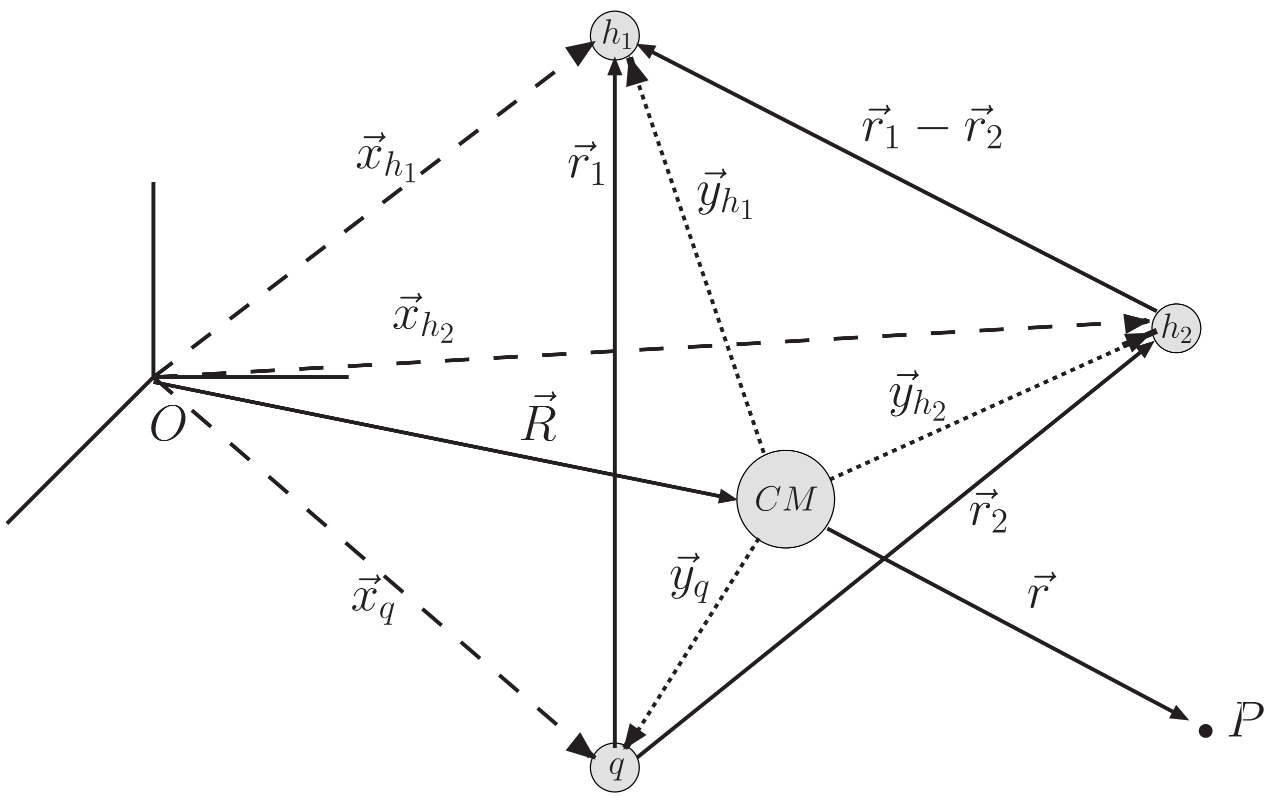
Baryon	Quark content	S_h	J^π	Baryon	Quark content	S_h	J^π
Ξ_{cc}	c c l	1	$1/2^+$	Ω_{cc}	c c s	1	$1/2^+$
Ξ_{cc}^*	c c l	1	$3/2^+$	Ω_{cc}^*	c c s	1	$3/2^+$
Ξ_{bb}	b b l	1	$1/2^+$	Ω_{bb}	b b s	1	$1/2^+$
Ξ_{bb}^*	b b l	1	$3/2^+$	Ω_{bb}^*	b b s	1	$3/2^+$
Ξ_{bc}	b c l	1	$1/2^+$	Ω_{bc}	b c s	1	$1/2^+$
Ξ_{bc}^*	b c l	1	$3/2^+$	Ω_{bc}^*	b c s	1	$3/2^+$
Ξ_{bc}'	b c l	0	$1/2^+$	Ω_{bc}'	b c s	0	$1/2^+$

l=u,d

Three–body Hamiltonian

$$H = -\frac{\vec{\nabla}^2_{\vec{R}}}{2\overline{M}} + H_{int} \qquad H_{int} = \overline{M} + \sum_{j=1,2} H_j^{sp} + V_{h_1,h_2}(\vec{r}_1 - \vec{r}_2, spin) - \frac{\vec{\nabla}_1 \cdot \vec{\nabla}_2}{m_q}$$

$$H_j^{sp} = -\frac{\vec{\nabla}_j^2}{2\mu_j} + V_{h_j,q}(\vec{r}_j, spin) \qquad \overline{M} = m_{h_1} + m_{h_2} + m_q \qquad \frac{1}{\mu_j} = \frac{1}{m_{h_j}} + \frac{1}{m_q}$$



Quark–quark potentials

We use 5 different quark–quark potentials taken from

C. Semay, B. Silvestre-Brac, Z. Phys. C**61**, 271 (1994)

R.K. Bhaduri, L.E. Cohler, Y. Nogami, Nuovo Cimento A**65**, 376 (1981)

Common structure:

- Confinement term
- OGE
  - Coulomb term
  - Hyperfine terms

$$V_{ij}^{q\bar{q}}(r) = -\frac{\kappa\left(1-e^{-r/r_c}\right)}{r} + \lambda r^p - \Lambda$$

$$+ \left\{ a_0 \frac{\kappa}{m_i m_j} \frac{e^{-r/r_0}}{r r_0^2} + \frac{2\pi}{3 m_i m_j} \kappa' \left(1 - e^{-r/r_c}\right) \frac{e^{-r^2/x_0^2}}{\pi^{\frac{3}{2}} x_0^3} \right\} \vec{\sigma}_i \vec{\sigma}_j$$

$V_{ij}^{qq}(r) = \frac{1}{2} V_{ij}^{q\bar{q}}(r)$ assuming a global $\vec{\lambda}_i^c \cdot \vec{\lambda}_j^c$ dependence

Differences:

- $p = 1$ (lattice QCD) or $p = 2/3$ (Regge trajectories)
- Form factors in OGE terms

Parameters adjusted to light and heavy-light meson spectroscopy

Baryon wave function

We are interested in ground state baryons

- $\Xi_{h_1h_2}, \Omega_{h_1h_2}$:

$$\sum_{M_{S_h} M_{S_l}} \left(1 \frac{1}{2} \frac{1}{2} \middle| M_{S_h} M_{S_l} M_J \right) |h_1 h_2; 1 M_{S_h}\rangle_{S_h} \otimes \left| q; \frac{1}{2} M_{S_l} \right\rangle \phi_{h_1 h_2 q}(r_1, r_2, r_{12})$$

- $\Xi^*_{h_1h_2}, \Omega^*_{h_1h_2}$:

$$\sum_{M_{S_h} M_{S_l}} \left(1 \, \frac{1}{2} \frac{3}{2} \, \middle| M_{S_h} \; M_{S_l} M_J \right) |h_1 h_2; 1 M_{S_h}\rangle_{S_h} \otimes \left| q; \frac{1}{2} M_{S_l} \right\rangle \phi_{h_1 h_2 q}(r_1, r_2, r_{12})$$

- $\Xi'_{h_1h_2}, \Omega'_{h_1h_2}$:

$$|h_1 h_2; 00\rangle_{S_h} \otimes \left| q; \frac{1}{2} M_{S_l} \right\rangle \phi_{h_1 h_2 q}(r_1, r_2, r_{12})$$

Variational Ansatz

- Use a simple ansatz for spatial wave function:

$$\phi_{h_1,h_2,q}(r_1,r_2,r_{12}) = N\phi^q_{h_1}(r_1)\phi^q_{h_2}(r_2)F(r_{12})$$

  - $\phi^q_{h_j}(r_j) = (1+\alpha_j\, r_j)\,\psi^q_{h_j}(r_j)$
    Ground state wave function $\psi^q_{h_j}(r_j)$ for the relative motion of the light–quark heavy–quark system for the chosen potential, corrected at large distances.
  - Jastrow correlation function

$$F(r_{12}) = \sum_{j=1}^{4} a_j\, e^{-b_j^2\,(r_{12}+d_j)^2} \qquad\qquad a_1 = 1$$

  - N: Normalization constant
- Parameters of the variational ansatz adjusted so that $\langle B\,|H|\,B\rangle$ is minimized

Spectra I

	This work	FADDEEV	RQM	PA SRQCD	BS	NRQM	RQPM	NRQM FH-SEMF
Ξ_{cc}	3612^{+17}	3607^{+24}	3620	3480	3740	3478	3660	3660 ± 70
Ξ^*_{cc}	3706^{+23}		3727	3610	3860	3610	3810	3740 ± 80
Ξ_{bb}	10197^{+10}_{-17}	10194^{+10}_{-19}	10202	10090	10300	10093	10230	10340 ± 100
Ξ^*_{bb}	10236^{+9}_{-17}		10237	10130	10340	10133	10280	10370 ± 100
Ξ_{bc}	6919^{+17}_{-7}	6915^{+17}_{-9}	6933	6820	7010	6820	6950	$6965 \pm 90^{\dagger}$
Ξ'_{bc}	6948^{+17}_{-6}		6963	6850	7070	6850	7000	$7065 \pm 90^{\dagger}$
Ξ^*_{bc}	6986^{+14}_{-5}		6980	6900	7100	6900	7020	7060 ± 90

FADDEEV, B. Silvestre-Brac,Few-Body Syst. 20, 1 (1996)
RQM, D. Ebert et al., Phys. Rev. D 66, 014008 (2002)
PA SRQCD, V.V. Kiselev and A.K. Likhoded, Phys. Usp. 45, 455 (2002)
BS, S.-P. Tong et al., Phys. Rev. D 62, 054024 (2000)
NRQM, S.S. Gershtein et al., Phys. Rev. D 62 054021 (2000)
RQPM, D. Ebert et al., Z. Phys. C 76, 111 (1997)
NRQM FE-SEMF, R. Roncaglia et al., Phys. Rev D 52, 1772 (1995), Phys. Rev. D 51, 1248 (1995)

Spectra II

	This work	FADDEEV	RQM	PA SRQCD	BS	NRQM	RQPM	NRQM FH-SEMF
Ω_{cc}	3702^{+41}	3710^{+29}_{-2}	3778	3590	3760	3590	3760	3740 ± 70
Ω^*_{cc}	3783^{+22}		3872	3690	3900	3690	3890	3820 ± 80
Ω_{bb}	10260^{+14}_{-34}	10267^{+4}_{-43}	10359	10180	10340	10180	10320	10370 ± 100
Ω^*_{bb}	10297^{+5}_{-28}		10389	10200	10380	10200	10360	10400 ± 100
Ω_{bc}	6986^{+27}_{-17}	7003^{+20}_{-32}	7088	6910	7050	6910	7050	$7045 \pm 90^{\dagger}$
Ω'_{bc}	7009^{+24}_{-15}		7116	6930	7110	6930	7090	$7105 \pm 90^{\dagger}$
Ω^*_{bc}	7046^{+11}_{-9}		7130	6990	7130	6990	7110	7120 ± 90

Spectra III

	This work	Exp.*	Latt. 1	Latt. 2	Latt. 3
Ξ_{cc}	3612^{+17}	3519 ± 1		3605 ± 23	3549 ± 95
Ξ^*_{cc}	3706^{+23}			3685 ± 23	3641 ± 97
Ξ_{bb}	10197^{+10}_{-17}		10314 ± 47		
Ξ^*_{bb}	10236^{+9}_{-17}		10333 ± 55		

	This work	Latt. 1	Latt. 2	Latt. 3
Ω_{cc}	3702^{+41}		$3733 \pm 9^{+7}_{-38}$	3663 ± 97
Ω^*_{cc}	3783^{+22}		$3801 \pm 9^{+3}_{-34}$	3734 ± 98
Ω_{bb}	10260^{+14}_{-34}	$10365 \pm 40^{-11}_{+12}{}^{+16}_{-0}$		
Ω^*_{bb}	10297^{+5}_{-28}	$10383 \pm 39^{-8}_{+8}{}^{+12}_{-0}$		

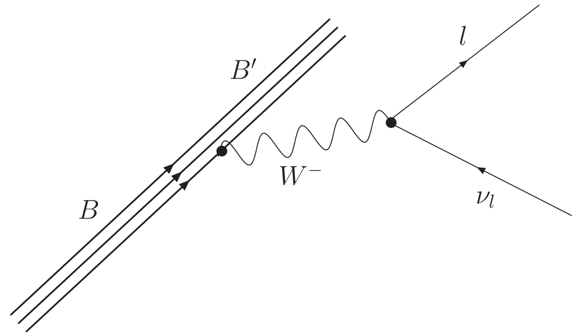
Exp.*, SELEX Coll. (M. Mattson et al.), Phys. Rev. Lett. 89, 112001 (2002)
Latt. 1, A. Ali Khan et al., Phys. Rev. D 62, 054505 (2000)
Latt. 2, R. Lewis et al., Phys. Rev. D 64, 094509 (2001)
Latt. 3, UKQCD Coll. (J.M. Flynn et al.), JHEP 0307, 066 (2003)

Spectra IV

	This work	RQM	PA SRQCD	BS	NRQM	RQPM	Latt. NRQCD	Latt.
$M_{\Xi^*_{cc}} - M_{\Xi_{cc}}$	94^{+5}_{-11}	107	130	120	132	150	70 ± 13	80 ± 11 [2]
								87 ± 19 [3]
$M_{\Xi^*_{bb}} - M_{\Xi_{bb}}$	39^{+1}_{-6}	35	40	40	40	50	20 ± 7	20 ± 6 [1]
$M_{\Xi^*_{bc}} - M_{\Xi_{bc}}$	67^{+3}_{-10}	47	80	90	80	70	43 ± 11	
$M_{\Xi'_{bc}} - M_{\Xi_{bc}}$	29^{+1}_{-5}	30	30	60	30	50	9 ± 7	
$M_{\Omega^*_{cc}} - M_{\Omega_{cc}}$	81^{+11}_{-19}	94	100	140	100	130	63 ± 9	68 ± 7 [2]
								67 ± 16 [3]
$M_{\Omega^*_{bb}} - M_{\Omega_{bb}}$	37^{+6}_{-9}	30	20	40	20	40	19 ± 5	20 ± 5 [1]
$M_{\Omega^*_{bc}} - M_{\Omega_{bc}}$	60^{+8}_{-16}	42	80	80	80	60	39 ± 8	
$M_{\Omega'_{bc}} - M_{\Omega_{bc}}$	23^{+2}_{-3}	28	20	60	20	40	9 ± 6	

Latt. NRQCD, N. Marthur et al., Phys. Rev. D 66, 014502 (2002)

Semileptonic decay ($b \to c$) I



- We only study $1/2 \to 1/2$ transitions:

$$\left\langle B', r'\vec{p}\right| (j_{bc})^\alpha \left|B, r\vec{p}\right\rangle \;=\; \bar{u}^{B'}_{r'}(\vec{p}\,')\left[\gamma^\alpha (F_1 - \gamma_5 G_1) \; + v^\alpha (F_2 - \gamma_5 G_2) + v'^\alpha (F_3 - \gamma_5 G_3)\right] u^B_r(\vec{p})$$

- Decay width can be written as $\Gamma = \Gamma_T + \Gamma_L$: $\qquad (w = v \cdot v')$

$$\frac{\mathrm{d}\Gamma_T}{\mathrm{d}w} = \frac{G^2|V_{cb}|^2}{12\pi^3} M_F^3 \sqrt{w^2-1}\, q^2 \left\{ (w-1)|F_1(w)|^2 + (w+1)|G_1(w)|^2 \right\}$$

$$\frac{\mathrm{d}\Gamma_L}{\mathrm{d}w} = \frac{G^2|V_{cb}|^2}{24\pi^3} M_F^3 \sqrt{w^2-1} \left\{ (w-1)|\mathcal{F}^V(w)|^2 + (w+1)|\mathcal{F}^A(w)|^2 \right\}$$

$$\mathcal{F}^{V,A}(w) = \left[(M_I \pm M_F) F_1^{V,A} + (1 \pm w)\left(M_F F_2^{V,A} + M_I F_3^{V,A} \right) \right]$$

$$F_i^V \equiv F_i(w); \qquad F_i^A \equiv G_i(w); \qquad i = 1, 2, 3$$

Semileptonic decay ($b \to c$) II

- Calculating Matrix elements (Initial Baryon Rest Frame)

$$\sqrt{\frac{E_{B'}(-\vec{q})}{m_{B'}}}\ {}_{NR}\left\langle B', r' \ -\vec{q}\left|\ \overline{\Psi}^c(0)\gamma_\mu(I-\gamma_5)\Psi^b(0)\right|B, r\ \vec{0}\right\rangle_{NR}$$

$$= 2\sqrt{\frac{E_{B'}(-\vec{q})}{m_{B'}}}\sum_{s_1}\sum_{s_2}\left(\frac{1}{2}\frac{1}{2}S_h\middle|s_1, s_2 - s_1, s_2\right)\left(S_h\frac{1}{2}\frac{1}{2}\middle|s_2, r - s_2, r\right)$$

$$\times\left(\frac{1}{2}\frac{1}{2}S'_h\middle|r' - r + s_1, s_2 - s_1, r' - r + s_2\right)\left(S'_h\frac{1}{2}\frac{1}{2}\middle|r' - r + s_2, r - s_2, r'\right)$$

$$\times\int d^3r_1\, d^3r_2\, \left(\Phi^{B'}_{ch_2q}(r_1, r_2, r_{12})\right)^*\ e^{i\frac{m_{h_2}}{\overline{M}'}\ \vec{q}\cdot\vec{r}_2}\ e^{-i\frac{m_{h_2}+m_q}{\overline{M}'}\ \vec{q}\cdot\vec{r}_1}$$

$$\times\sqrt{\frac{m_b}{E_b(\vec{l}\,)}}\sqrt{\frac{m_c}{E_c(\vec{l}-\vec{q})}}\ \bar{u}^c_{r'-r+s_1}(\vec{l}-\vec{q})\gamma_\mu(I-\gamma_5)\, u^b_{s_1}(\vec{l}\,)\ \Phi^B_{bh_2q}(r_1, r_2, r_{12})$$

with $\vec{l} = -i\vec{\nabla}_{r_1}$

- Approximation:
  - First order in $\vec{l}$
  - Keep all orders on transferred momentum $\vec{q}$.

Semileptonic decay ($b \to c$) III

All form factors can be evaluated in terms of two integrals

$$I = \int d^3 r_1 d^3 r_2 e^{i\vec{q}\left(\frac{m_{h_2}}{\overline{M}'}\vec{r}_2 - \frac{m_{h_2}+m_q}{\overline{M}'}\vec{r}_1\right)} (\Phi^{B'}_{ch_2q}(r_1, r_2, r_{12}))^* \Phi^{B}_{bh_2q}(r_1, r_2, r_{12})$$

$$K = \frac{1}{|\vec{q}|^2} \int d^3 r_1 d^3 r_2 e^{i\vec{q}\left(\frac{m_{h_2}}{\overline{M}'}\vec{r}_2 - \frac{m_{h_2}+m_q}{\overline{M}'}\vec{r}_1\right)} (\Phi^{B'}_{ch_2q}(r_1, r_2, r_{12}))^* \vec{l}\vec{q}\, \Phi^{B}_{bh_2q}(r_1, r_2, r_{12})$$

Example: $\Xi'_{bc} \to \Xi_{cc}\, l\, \bar{\nu}_l$

- Vector part

$$\hat{F}_1 + \hat{F}_2 + \frac{E_{B'}}{M_{B'}}\hat{F}_3 = 0$$

$$\frac{1}{E_{B'} + M_{B'}}\hat{F}_1 + \frac{1}{M_{B'}}\hat{F}_3 = 0$$

$$\frac{1}{E_{B'} + M_{B'}}\hat{F}_1 = \frac{-2}{\sqrt{3}}\left\{\frac{I}{E_c + m_c} - \frac{K}{2}\left(\frac{m_c}{E_c^2} - \frac{1}{m_b}\right)\right\}$$

$$E_c^2 = \vec{q}^{\,2} + m_c^2$$

$$\hat{F}_i^{V,A} = \sqrt{\frac{2E_c}{E_c+m_c}}\sqrt{\frac{M_{B'}}{E_{B'}}}\sqrt{\frac{E_{B'}+M_{B'}}{2E_{B'}}}F_i^{V,A}$$

- Axial part

$$\frac{1}{E_{B'} + M_{B'}}\left(-\hat{G}_1 + \hat{G}_2 + \frac{E_{B'}}{M_{B'}}\hat{G}_3\right) = \frac{-2}{\sqrt{3}}\left\{\frac{-I}{E_c + m_c} + \frac{K}{2}\left(\frac{m_c}{E_c^2} + \frac{1}{m_b}\right)\right\}$$

$$\hat{G}_1 - \frac{\vec{q}^{\,2}}{M_{B'}\left(E_{B'} + M_{B'}\right)}\hat{G}_3 = \frac{-2}{\sqrt{3}}\left\{I + \frac{\vec{q}^{\,2}}{2\left(E_c + m_c\right)}K\left(\frac{m_c}{E_c^2} - \frac{1}{m_b}\right)\right\}$$

$$\hat{G}_1 = \frac{-2}{\sqrt{3}}\left\{I + \frac{\vec{q}^{\,2}}{2\left(E_c + m_c\right)}K\left(\frac{m_c}{E_c^2} + \frac{1}{m_b}\right)\right\}$$

Semileptonic decay widths I

	Γ_T	Γ_L	Γ
$\Xi_{bb} \to \Xi_{bc}\, l\bar{\nu}_l$	$0.97^{+0.10}_{-0.02}$	$1.28^{+0.19}_{-0.04}$	$2.25^{+0.29}_{-0.06}$
$\Xi_{bc} \to \Xi_{cc}\, l\bar{\nu}_l$	$1.15^{+0.08}_{-0.01}$	$1.86^{+0.}_{-0.02}$	$3.01^{+0.30}_{-0.03}$
$\Xi_{bb} \to \Xi'_{bc}\, l\bar{\nu}_l$	$0.73^{+0.08}_{-0.02}$	$0.52^{+0.07}_{-0.01}$	$1.24^{+0.15}_{-0.03}$
$\Xi'_{bc} \to \Xi_{cc}\, l\bar{\nu}_l$	$0.89^{+0.05}_{-0.02}$	$0.70^{+0.06}_{-0.01}$	$1.59^{+0.11}_{-0.03}$
$\Omega_{bb} \to \Omega_{bc}\, l\bar{\nu}_l$	$1.06^{+0.07}_{-0.01}$	$1.45^{+0.16}_{-0.01}$	$2.51^{+0.23}_{-0.02}$
$\Omega_{bc} \to \Omega_{cc}\, l\bar{\nu}_l$	$1.15^{+0.06}$	$1.88^{+0.17}$	$3.03^{+0.23}$
$\Omega_{bb} \to \Omega'_{bc}\, l\bar{\nu}_l$	$0.79^{+0.08}$	$0.57^{+0.07}$	$1.36^{+0.15}$
$\Omega'_{bc} \to \Omega_{cc}\, l\bar{\nu}_l$	$0.89^{+0.05}$	$0.70^{+0.05}$	$1.59^{+0.10}$

In units of $|V_{cb}|^2\, 10^{-11}$ MeV

Semileptonic decay widths II

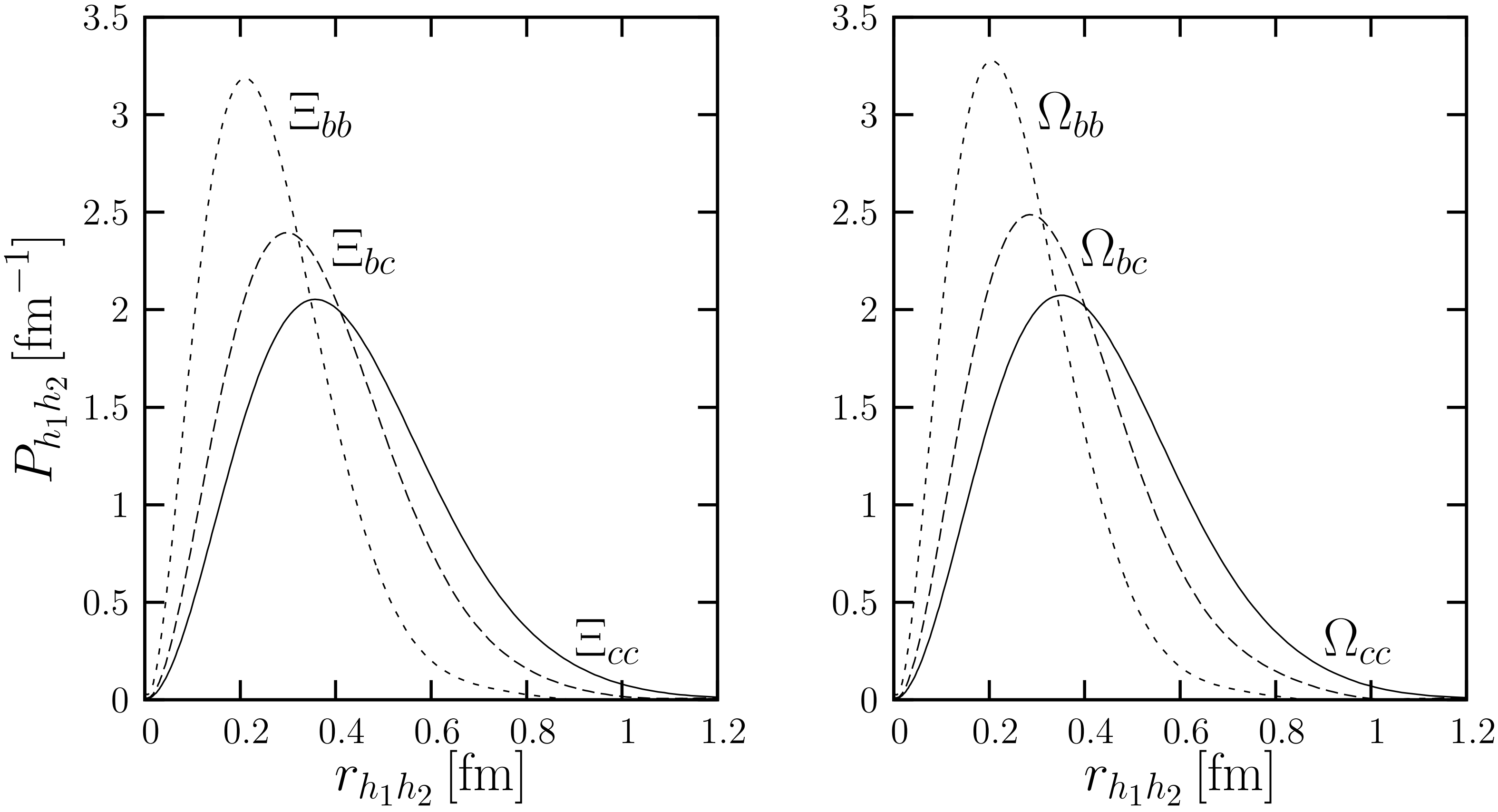
	This work	RQM	RTQM	BS	HQET
$\Gamma(\Xi_{bb} \to \Xi_{bc}\, l\bar{\nu}_l)$	$3.84^{+0.49}_{-0.10}$	3.26		28.5	
$\Gamma(\Xi_{bc} \to \Xi_{cc}\, l\bar{\nu}_l)$	$5.13^{+0.51}_{-0.05}$	4.59	0.79	8.93	4.0
$\Gamma(\Xi_{bb} \to \Xi'_{bc}\, l\bar{\nu}_l)$	$2.12^{+0.26}_{-0.05}$	1.64		4.28	
$\Gamma(\Xi'_{bc} \to \Xi_{cc}\, l\bar{\nu}_l)$	$2.71^{+0.19}_{-0.05}$	1.76		7.76	
$\Gamma(\Omega_{bb} \to \Omega_{bc}\, l\bar{\nu}_l)$	$4.28^{+0.39}_{-0.03}$	3.40		28.8	
$\Gamma(\Omega_{bc} \to \Omega_{cc}\, l\bar{\nu}_l)$	$5.17^{+0.39}$	4.95			
$\Gamma(\Omega_{bb} \to \Omega'_{bc}\, l\bar{\nu}_l)$	$2.32^{+0.26}$	1.66			
$\Gamma(\Omega'_{bc} \to \Omega_{cc}\, l\bar{\nu}_l)$	$2.71^{+0.17}$	1.90			

In units of 10^{-14} GeV. We have used a value $|V_{cb}| = 0.0413$.

RQM, D. Ebert et al., Phys. Rev. D 70, 014018 (2004)
RTQM, A. Faessler et al., Phys. Lett. B 518, 55 (2001)
BS,X.-H. Guo et al., Phys. Rev. D 58, 114007 (1998)
HQET,M.A. Sanchis-Lozano, Nucl. Phys. B 440, 251 (1995)

Infinite heavy quark mass limit I

“In the infinite heavy quark mass limit doubly heavy baryons should look like a meson composed of a light quark and a heavy diquark that to the light quark degrees of freedom appears to be pointlike’.’



Infinite heavy quark mass limit II

The wave function should reduce in that limit to the product $\Phi_{h_1h_2}(r_{12}) \cdot \Phi^0_{qh_1h_2}(r_q)$, being $\Phi_{h_1h_2}(r_{12}),\ \Phi^0_{qh_1h_2}(r_q)$ the ground state wave functions for $H_{h_1h_2},\ H^0_{qh_1h_2}$ with

$$
\begin{aligned}
H_{h1h_2} &= -\frac{\vec{\nabla}^2_{12}}{2\mu_{h_1h_2}} + V_{h_1h_2}(\vec{r}_{12},\ spin) \\
H^0_{qh_1h_2} &= -\frac{1}{2}\left(\frac{1}{m_{h_1}+m_{h_2}} + \frac{1}{m_q}\right)\vec{\nabla}^2_q + V_{h_1q}(\vec{r}_q,\ spin) + V_{h_2q}(\vec{r}_q,\ spin)
\end{aligned}
$$

where $\mu_{h_1h_2} = m_{h_1}m_{h_2}/(m_{h_1}+m_{h_2}),\ \vec{r}_q = (m_{h_1}\vec{r}_1 + m_{h_2}\vec{r}_2)/(m_{h_1}+m_{h_2})$
Evaluating

$$
\mathcal{P} = \int d^3r_1 \int d^3r_2 \left(\Phi^B_{h_1h_2q}(r_1, r_2, r_{12})\right)^* \Phi_{h_1h_2}(r_{12})\ \Phi^0_{qh_1h_2}(r_q)
$$

	Ξ_{cc}	Ξ_{bc}	Ξ_{bb}	Ω_{cc}	Ω_{bc}	Ω_{bb}
$\|\mathcal{P}\|^2$	0.974	0.975	0.991	0.959	0.966	0.984

Summary and Outlook

- To solve the three-body problem we have used a simple variational ansatz made possible by HQSS constraints
- Our wave functions comply with the infinite heavy quark mass limit
- Spectrum
  - Results stable against different quark-quark potentials $(\sim 1\%)$
  - Results in agreement with previous calculations
  - Good agreement with latice data for cc baryons
- Semileptonic decays:
  - Results rather stable against different potentials
  - Agreement with the RQM calculation of Ebert et al. but clear disagreement with other calculations
- Check the form factors infinite heavy quark mass limit